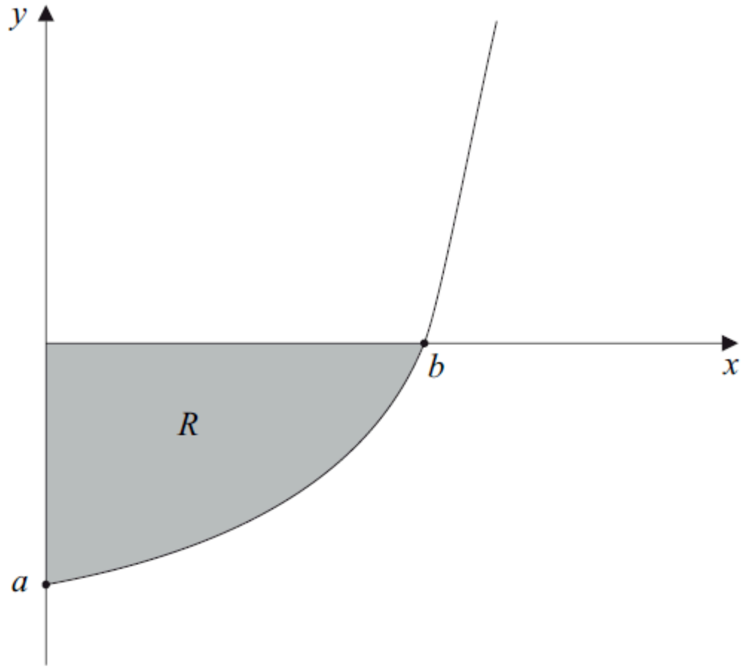
Core 3 Exponentials & Logarithms Questions

5 The diagram shows part of the graph of $y = e^{2x} - 9$. The graph cuts the coordinate axes at $(0, a)$ and $(b, 0)$.



(a) State the value of a, and show that $b = \ln 3$. *(3 marks)*

(b) Show that $y^2 = e^{4x} - 18e^{2x} + 81$. *(1 mark)*

(c) The shaded region R is rotated through 360° about the x-axis. Find the volume of the solid formed, giving your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers. *(6 marks)*

(d) Sketch the curve with equation $y = |e^{2x} - 9|$ for $x \geqslant 0$. *(2 marks)*

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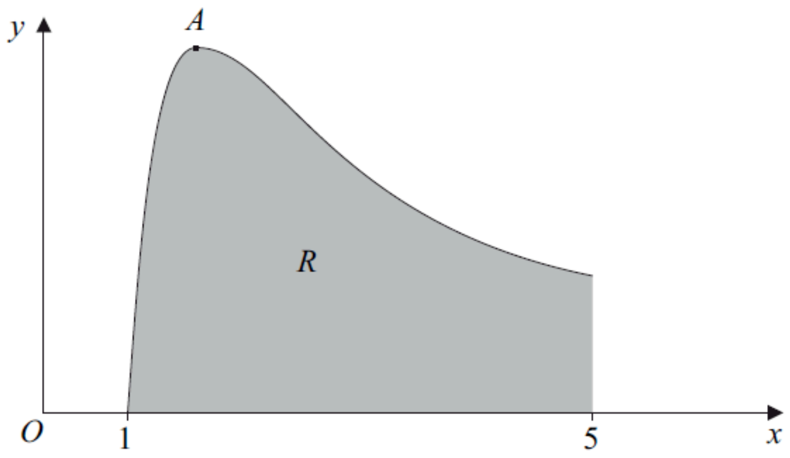
9 (a) Given that $y = x^{-2} \ln x$, show that $\dfrac{dy}{dx} = \dfrac{1 - 2\ln x}{x^3}$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2} \ln x$.



(i) Using the answer to part (a), find, in terms of e, the x-coordinate of the stationary point A. *(2 marks)*

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5 (a) A curve has equation $y = e^{2x} - 10e^{x} + 12x$.

(i) Find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Find $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(b) The points P and Q are the stationary points of the curve.

(i) Show that the x-coordinates of P and Q are given by the solutions of the equation

$$e^{2x} - 5e^{x} + 6 = 0$$ *(1 mark)*

(ii) By using the substitution $z = e^{x}$, or otherwise, show that the x-coordinates of P and Q are $\ln 2$ and $\ln 3$. *(3 marks)*

(iii) Find the y-coordinates of P and Q, giving each of your answers in the form $m + 12 \ln n$, where m and n are integers. *(3 marks)*

(iv) Using the answer to part (a)(ii), determine the nature of each stationary point.

(b) (i) Given that $y = x \ln x$, find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Hence, or otherwise, find $\displaystyle\int \ln x \, dx$. *(2 marks)*

(iii) Find the exact value of $\displaystyle\int_1^5 \ln x \, dx$. *(2 marks)*

(b) (i) Find $\dfrac{dx}{dy}$ when $x = 2y^3 + \ln y$. *(1 mark)*

(ii) Hence find an equation of the tangent to the curve $x = 2y^3 + \ln y$ at the point $(2,1)$. *(3 marks)*

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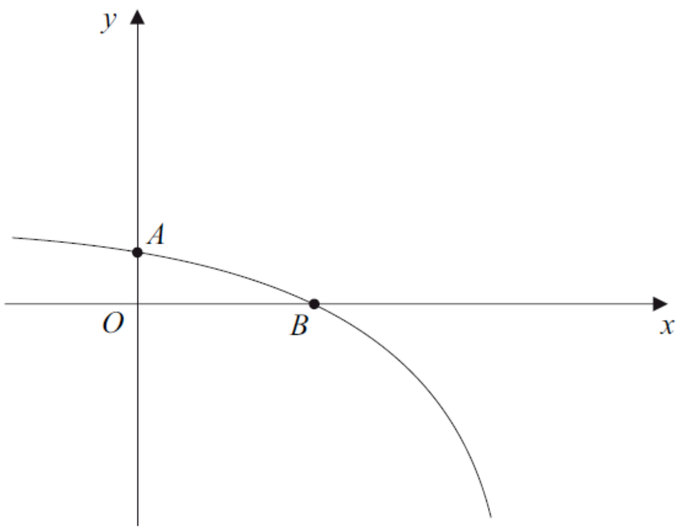
9 The sketch shows the graph of $y = 4 - e^{2x}$. The curve crosses the y-axis at the point A and the x-axis at the point B.



(a) (i) Find $\int (4 - e^{2x}) dx$. *(2 marks)*

(ii) Hence show that $\int_0^{\ln 2} (4 - e^{2x}) dx = 4\ln 2 - \frac{3}{2}$. *(2 marks)*

(b) (i) Write down the y-coordinate of A. *(1 mark)*

(ii) Show that $x = \ln 2$ at B. *(2 marks)*

(c) Find the equation of the normal to the curve $y = 4 - e^{2x}$ at the point B. *(4 marks)*

(d) Find the area of the region enclosed by the curve $y = 4 - e^{2x}$, the normal to the curve at B and the y-axis. *(3 marks)*

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1 (a) Differentiate $\ln x$ with respect to x. *(1 mark)*

(b) Given that $y = (x + 1)\ln x$, find $\frac{dy}{dx}$. *(2 marks)*

(c) Find an equation of the normal to the curve $y = (x + 1)\ln x$ at the point where $x = 1$.

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7 (a) A curve has equation $y = (x^2 - 3)e^x$.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Find $\frac{d^2y}{dx^2}$. *(2 marks)*

(b) (i) Find the x-coordinate of each of the stationary points of the curve. *(4 marks)*

(ii) Using your answer to part (a)(ii), determine the nature of each of the stationary points. *(2 marks)*