Core 3 Integration Questions

3 (a) (i) Given that $f(x) = x^4 + 2x$, find $f'(x)$. *(1 mark)*

(ii) Hence, or otherwise, find $\int \frac{2x^3 + 1}{x^4 + 2x}\,dx$. *(2 marks)*

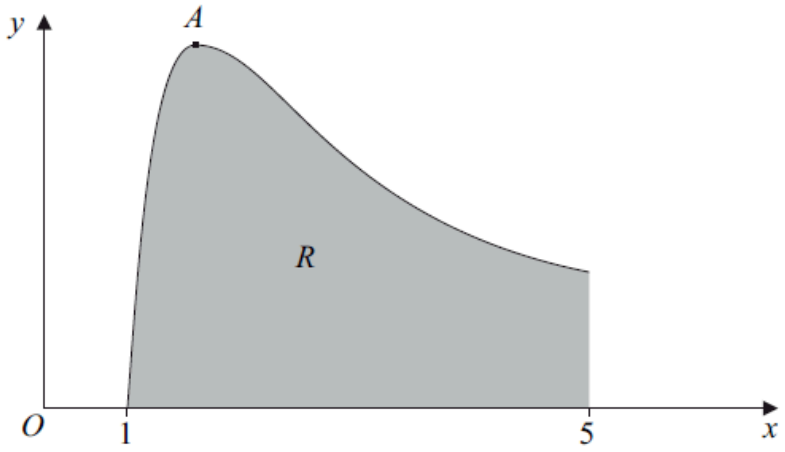
(b) (i) Use the substitution $u = 2x + 1$ to show that

$$\int x\sqrt{2x+1}\,dx = \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du \qquad \textit{(3 marks)}$$

(ii) Hence show that $\int_0^4 x\sqrt{2x+1}\,dx = 19.9$ correct to three significant figures. *(4 marks)*

(b) Using integration by parts, find $\int x^{-2}\ln x\,dx$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2}\ln x$.



(ii) The region R is bounded by the curve, the x-axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

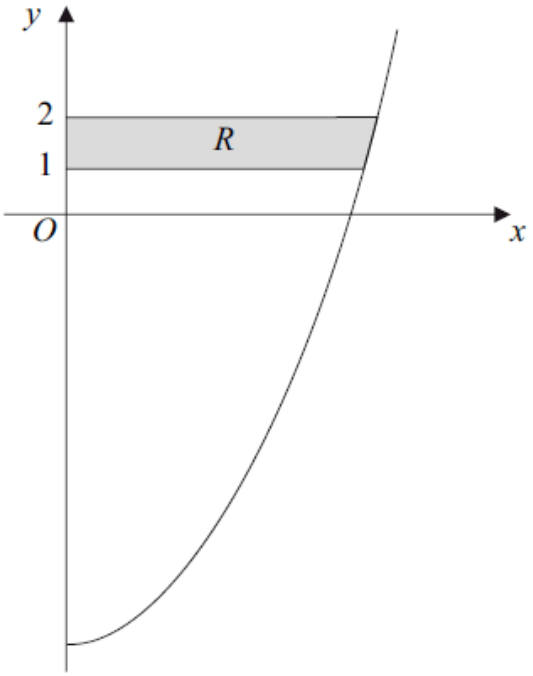
$$\frac{1}{5}(4 - \ln 5) \qquad \textit{(3 marks)}$$

(b) Use the substitution $u = 2x + 1$ to find $\int x(2x+1)^8\,dx$, giving your answer in terms of x. *(4 marks)*

4 (a) Use integration by parts to find $\int x \sin x \, dx$. *(4 marks)*

(b) Using the substitution $u = x^2 + 5$, or otherwise, find $\int x\sqrt{x^2 + 5} \, dx$. *(4 marks)*

(c) The diagram shows the curve $y = x^2 - 9$ for $x \geqslant 0$.



The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° **about the y-axis**. *(4 marks)*

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6 (a) Use integration by parts to find $\int xe^{5x} \, dx$. *(4 marks)*

(b) (i) Use the substitution $u = \sqrt{x}$ to show that

$$\int \frac{1}{\sqrt{x}(1 + \sqrt{x})} \, dx = \int \frac{2}{1 + u} \, du$$ *(2 marks)*

(ii) Find the exact value of $\int_1^9 \frac{1}{\sqrt{x}(1 + \sqrt{x})} \, dx$. *(3 marks)*
