Core 3 Trigonometry Questions

1 (a) Find $\frac{dy}{dx}$ when $y = \tan 3x$. *(2 marks)*

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4 It is given that $2\text{cosec}^2 x = 5 - 5\cot x$.

(a) Show that the equation $2\text{cosec}^2 x = 5 - 5\cot x$ can be written in the form

$$2\cot^2 x + 5\cot x - 3 = 0$$ *(2 marks)*

(b) Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$. *(2 marks)*

(c) Hence, or otherwise, solve the equation $2\text{cosec}^2 x = 5 - 5\cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leqslant \pi$. *(3 marks)*

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3 (a) Solve the equation $\sec x = 5$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(3 marks)*

(b) Show that the equation $\tan^2 x = 3\sec x + 9$ can be written as

$$\sec^2 x - 3\sec x - 10 = 0$$ *(2 marks)*

(c) Solve the equation $\tan^2 x = 3\sec x + 9$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

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7 (a) Given that $z = \frac{\sin x}{\cos x}$, use the quotient rule to show that $\frac{dz}{dx} = \sec^2 x$. *(3 marks)*

(b) Sketch the curve with equation $y = \sec x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. *(2 marks)*

(b) (i) Given that $y = \sin^{-1} 2x$, show that $x = \frac{1}{2}\sin y$. *(1 mark)*

(ii) Given that $x = \frac{1}{2}\sin y$, find $\frac{dx}{dy}$ in terms of y. *(1 mark)*

(c) Using the answers to part (b) and a suitable trigonometrical identity, show that

$$\frac{dy}{dx} = \frac{2}{\sqrt{1 - 4x^2}}$$ *(4 marks)*

2 Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sec x$ onto the graph of $y = 1 + \sec 3x$. *(4 marks)*

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5 (a) (i) Show that the equation

$$2\cot^2 x + 5\operatorname{cosec} x = 10$$

can be written in the form $2\operatorname{cosec}^2 x + 5\operatorname{cosec} x - 12 = 0$. *(2 marks)*

(ii) Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$. *(3 marks)*

(b) Hence, or otherwise, solve the equation

$$2\cot^2(\theta - 0.1) + 5\operatorname{cosec}(\theta - 0.1) = 10$$

giving all values of θ in radians to two decimal places in the interval $-\pi < \theta < \pi$. *(3 marks)*

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6 (a) Find $\frac{dy}{dx}$ when:

(ii) $y = x^2 \tan x$. *(2 marks)*

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8 (a) Write down $\int \sec^2 x \, dx$. *(1 mark)*

(b) Given that $y = \frac{\cos x}{\sin x}$, use the quotient rule to show that $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. *(4 marks)*

(c) Prove the identity $(\tan x + \cot x)^2 = \sec^2 x + \operatorname{cosec}^2 x$. *(3 marks)*

(d) Hence find $\int_{0.5}^{1} (\tan x + \cot x)^2 \, dx$, giving your answer to two significant figures. *(4 marks)*