Core 4 Exponential Growth Answers

4(a)	$A = 80$	B1	1	
(b)	$5000 = 80 \times k^{56}$	M1		SC1 Verification. Need 62.51 or better
	$k = \sqrt[56]{\frac{5000}{80}} \approx 1.07664$	M1A1	3	**Or** using logs: M1 $\ln\left(\frac{5000}{80}\right) = 56\ln k$ A1 $k = e^{\ln\left(\frac{62.5}{56}\right)}$ **Or** 3/3 for $k = 1.076636$ **Or** 1.076637 seen
(c)(i)	$V = 80 \times k^{106} = 200707$	M1A1	2	200648 using full register k
(ii)	$\ln 10000 = \ln k^t$	M1		
	$t = \frac{\ln 10000}{\ln k} = 124.7 \Rightarrow 2024$	M1A1	3	M1 $t \ln k = \ln 10000$ A1 CAO **Or** trial and improvement M1expression M1 125, 124, A1 2024
	Total		9	

8(a)(i)	$(5000 - x)$ seen in a product	B1		Could be implied, eg $5000a - xa$
	$\frac{dx}{dt} = kx(5000 - x)$	B1	2	
(ii)	$200 = k \times 1000 \times (5000 - 1000)$	M1		$\frac{dx}{dt} = 200, x = 1000$ in their diff. equation Condone ts and $t = 0$ for M1
	$k = 0.00005$	A1	2	CAO OE
(b)(i)	$t = 4\ln\left(\frac{4 \times 2500}{5000 - 2500}\right) = 5.5$ (hours)	M1 A1	2	$x \to 2500$ (or $4 \ln 4$) CAO
(ii)	$e^{\frac{30}{4}}$	B1		
	$e^{7.5} = \frac{4x}{5000 - x}$	M1		OE
	$5000 \times e^{7.5} = x\left(4 + e^{7.5}\right)$	m1		Soluble for x
	$x = 4988.96.. \Rightarrow 4989$ rabbits infected	A1	4	Or 4988 or 4990; integer value only
	Total		10	

8(a)(i)	$\int \frac{dy}{y} = \int \sin t \, dt$	M1		Attempt to separate and integrate
	$\ln y = -\cos t + C$	A1,A1		A1 for $\ln y$; A1 for $-\cos t$; condone missing C
	$y = Ae^{-\cos t}$	A1	4	A present; or $y = e^{-\cos t + C}$
(ii)	$y = 50,\ t = \pi:\quad 50 = Ae^{-\cos\pi} = Ae$	M1 A1		Substitute $y = 50,\ t = \pi$ to find constant Can have $50 = e^{1+C}$ if substituted in above $e^C = \frac{50}{e}$
	$y = 50e^{-1}e^{-\cos t}$	A1	3	AG (convincingly obtained)
	Alternative: Must have a constant in answer to (a)(i) $y = Ae^{-\cos t}$ or $y = e^{-\cos t + c}$ or $\ln y = -\cos t + c$			**Alternative:** Substitute $y = 50,\ t = \pi$ into $\ln y = -\cos t + c$ M1 $\ln y = -\cos t + \ln 50 - 1$ A1
	$50 = Ae^{-\cos\pi}$ $\quad 50 = e^{-\cos\pi + c}$ $\quad \ln 50 = -\cos\pi + c$	(M1)		$\ln \frac{y}{50} = -1 - \cos t$ (AG) A1
	$50 = Ae$ $\quad 50 = e^{1+c}$ $\quad \ln y = -\cos t + \ln 50 - 1$	(A1)		
	$y = 50e^{-1-\cos t}$ $\quad y = e^{-\cos t}\frac{50}{e}$ $\quad \ln\left(\frac{y}{50}\right) = -1 - \cos t$	(A1)		
(b)(i)	$t = 6:\ y = 50e^{-1}e^{-\cos 6} = 7.0417\ldots \approx 7\text{cm}$	M1A1	2	Degrees 6.8 SC1 7 or 7.0 for A1
(ii)	$t = \pi \Rightarrow (\sin t = 0 \Rightarrow)\ \frac{dy}{dt} = 0$	B1		Condone x for t
	$\frac{d^2y}{dt^2} = y\cos t + \frac{dy}{dt}\sin t$	M1		For attempt at product rule including $\frac{dy}{dt}$ term; must have $\frac{d^2y}{dt^2} =$
		A1		
	$t = \pi$ $\frac{d^2y}{dt^2} = y\cos\pi + \frac{dy}{dt}\sin\pi$ $= -50 \Rightarrow$ max	A1	4	Accept $= -y$, with explanation that y is never negative
8(b)(ii) (cont)	**Alternative:** $y = 50e^{-(1+\cos t)} = \frac{50}{e}e^{-\cos t}$			
	$\frac{dy}{dt} = \frac{50}{e}e^{-\cos t} \times \sin t = 0$ at $t = \pi$	(B1)		
	$\frac{d^2y}{dt^2} = \frac{50}{e}e^{-\cos t} \times \cos t + \frac{50}{e}e^{-\cos t} \times \sin^2 t$	(M1) (A1)		Attempt at product rule Correct
	Substitute $t = \pi \to -50 \Rightarrow$ max	(A1)		
	Total		**13**	

4(a)(i)	$t=0:\ x=3$	B1	1	
(ii)	$t=14:\ x=15-12\mathrm{e}^{-1}$	M1		or $15-12\,\mathrm{e}^{\frac{-14}{14}}$
	$=10.6$	A1	2	CAO
(b)(i)	$-5=-12\mathrm{e}^{-\frac{t}{14}}$	M1		substitute $x=10$; rearrange to form $p=q\mathrm{e}^{-\frac{t}{14}}$
	$\ln\left(\frac{5}{12}\right)=-\frac{t}{14}$ (OE)	m1		take lns correctly
	$t=14\ln\left(\frac{12}{5}\right)$	A1	3	must come from correct working
(ii)	$t=12.256...\approx 12$ days	B1F	1	ft on a, b if $a>b$; accept $t=12$ NMS Accept 12 from incorrect working in b(i) Accept 13 if 12.2 or 12.3 seen
(c)(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{1}{14}\times-12\mathrm{e}^{-\frac{t}{14}}$	M1		differentiate; allow sign error condone $\frac{\mathrm{d}y}{\mathrm{d}x}$ used consistently
	$=-\frac{1}{14}(x-15)$	m1		Or $\frac{1}{14}\left(12\mathrm{e}^{-\frac{t}{14}}\right)$ and $12\mathrm{e}^{-\frac{t}{14}}=15-x$ seen
	$=\frac{1}{14}(15-x)$	A1	3	AG – be convinced CSO
	Alt: $t=-14\ln\left(\frac{15-x}{12}\right)$	(M1)		attempt to solve given equation for t
	$\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{-14\left(-\frac{1}{12}\right)}{\left(\frac{15-x}{12}\right)}$	(m1)		differentiate wrt x, with $\frac{1}{\frac{15-x}{12}}$ seen; OE
	$\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{14}{15-x}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{14}(15-x)$	(A1)	(3)	AG – be convinced
	Alt: (backwards) $\int\frac{\mathrm{d}x}{15-x}=\int\frac{\mathrm{d}t}{14}=\pm 14\ln(15-x)=t+c$	(M1)		
	Use $(0,3)$: $-14\ln(15-x)+14\ln 12=t$	(m1)		
	Solve for x: $x=15-12\mathrm{e}^{-\frac{t}{14}}$	(A1)	(3)	All steps shown
(ii)	rate of growth = 0.5 (cm per day)	B1	1	Accept $\frac{7}{14}$
	Total		**11**	