Core 4 Algebra & Functions Questions

1 (a) The polynomial $f(x)$ is defined by $f(x) = 3x^3 + 2x^2 - 7x + 2$.

    (i) Find $f(1)$. *(1 mark)*

    (ii) Show that $f(-2) = 0$. *(1 mark)*

    (iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax+b}$$

    where a and b are integers. *(3 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 3x^3 + 2x^2 - 7x + d$.

    When $g(x)$ is divided by $(3x - 1)$, the remainder is 2. Find the value of d. *(3 marks)*

(c) Given that $\dfrac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$ can be written in the form $\dfrac{A}{(3 - 2x)} + \dfrac{B}{(1 - x)} + \dfrac{C}{(1 - x)^2}$,

    find the values of A, B and C. *(5 marks)*

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1 (a) The polynomial $p(x)$ is defined by $p(x) = 6x^3 - 19x^2 + 9x + 10$.

    (i) Find $p(2)$. *(1 mark)*

    (ii) Use the Factor Theorem to show that $(2x + 1)$ is a factor of $p(x)$. *(3 marks)*

    (iii) Write $p(x)$ as the product of three linear factors. *(2 marks)*

(b) Hence simplify $\dfrac{3x^2 - 6x}{6x^3 - 19x^2 + 9x + 10}$. *(2 marks)*

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3 (a) Given that $\dfrac{9x^2 - 6x + 5}{(3x - 1)(x - 1)}$ can be written in the form $3 + \dfrac{A}{3x - 1} + \dfrac{B}{x - 1}$, where A and B are integers, find the values of A and B. *(4 marks)*

(b) Hence, or otherwise, find $\displaystyle\int \dfrac{9x^2 - 6x + 5}{(3x - 1)(x - 1)}\, dx$. *(4 marks)*

2 The polynomial $f(x)$ is defined by $f(x) = 2x^3 - 7x^2 + 13$.

(a) Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $(2x - 3)$.
(2 marks)

(b) The polynomial $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + 13 + d$, where d is a constant.

Given that $(2x - 3)$ is a factor of $g(x)$, show that $d = -4$.
(2 marks)

(c) Express $g(x)$ in the form $(2x - 3)(x^2 + ax + b)$.
(2 marks)

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4 (a) (i) Express $\dfrac{3x - 5}{x - 3}$ in the form $A + \dfrac{B}{x - 3}$, where A and B are integers.
(2 marks)

(ii) Hence find $\displaystyle\int \dfrac{3x - 5}{x - 3}\, dx$.
(2 marks)

(b) (i) Express $\dfrac{6x - 5}{4x^2 - 25}$ in the form $\dfrac{P}{2x + 5} + \dfrac{Q}{2x - 5}$, where P and Q are integers.
(3 marks)

(ii) Hence find $\displaystyle\int \dfrac{6x - 5}{4x^2 - 25}\, dx$.
(3 marks)

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1 (a) Find the remainder when $2x^2 + x - 3$ is divided by $2x + 1$.
(2 marks)

(b) Simplify the algebraic fraction $\dfrac{2x^2 + x - 3}{x^2 - 1}$.
(3 marks)

(b) Express $\dfrac{1 + 4x}{(1 + x)(1 + 3x)}$ in partial fractions.
(3 marks)