Decision 1 Matching Graph Questions

1 (a) Draw a bipartite graph representing the following adjacency matrix. *(2 marks)*

	U	***V***	***W***	***X***	***Y***	***Z***
A	1	0	1	0	1	0
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	0	0	1	0	1	1
F	0	0	0	1	1	0

(b) Given that initially A is matched to W, B is matched to X, C is matched to V, and E is matched to Y, use the alternating path algorithm, from this initial matching, to find a complete matching. List your complete matching. *(5 marks)*

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1 Five people, A, B, C, D and E, are to be matched to five tasks, 1, 2, 3, 4 and 5. The table shows which tasks each person can do.

Person	**Tasks**
A	1, 3, 5
B	2, 4
C	2
D	4, 5
E	3, 5

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially A is matched to task 3, B to task 4, C to task 2 and E to task 5.

Use an alternating path from this initial matching to find a complete matching. *(4 marks)*

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2 Five people A, B, C, D and E are to be matched to five tasks R, S, T, U and V.

The table shows the tasks that each person is able to undertake.

Person	**Tasks**
A	R, V
B	R, T
C	T, V
D	U, V
E	S, U

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task V, B to task R, C to task T, and E to task U.

Demonstrate, by using an alternating path from this initial matching, how each person can be matched to a task. *(4 marks)*

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1 Six people, A, B, C, D, E and F, are to be matched to six tasks, 1, 2, 3, 4, 5 and 6. The following adjacency matrix shows the possible matching of people to tasks.

	Task 1	**Task 2**	**Task 3**	**Task 4**	**Task 5**	**Task 6**
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	0	1	0	1	1
D	0	0	0	1	0	0
E	0	1	0	0	0	1
F	0	0	0	1	1	0

(a) Show this information on a bipartite graph. *(2 marks)*

(b) At first F insists on being matched to task 4. Explain why, in this case, a complete matching is impossible. *(1 mark)*

(c) To find a complete matching F agrees to be assigned to either task 4 or task 5.

Initially B is matched to task 3, C to task 6, E to task 2 and F to task 4.

From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching. *(6 marks)*