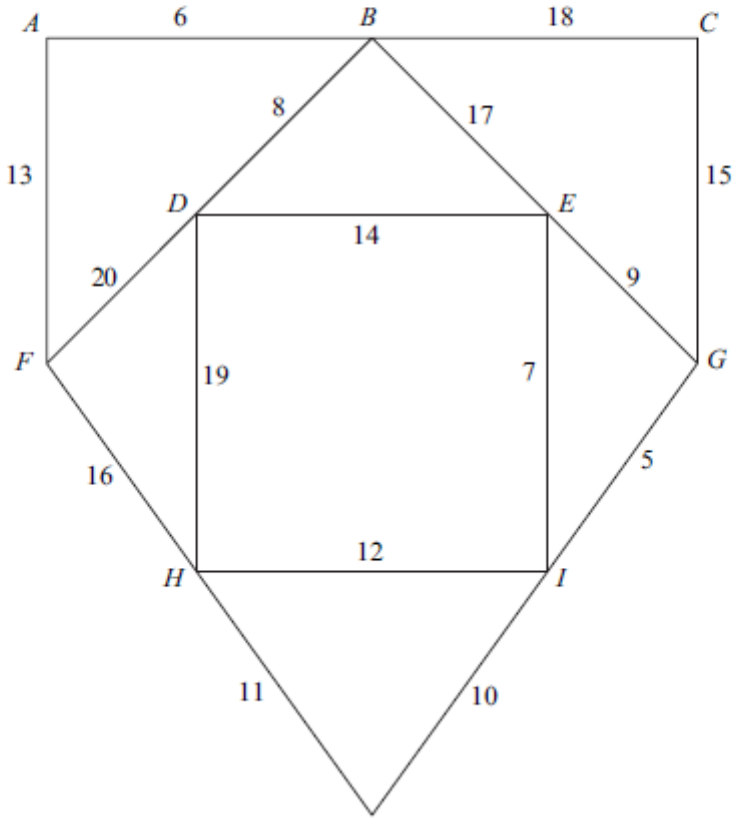
Decision 1 Minimum Spanning Tree Questions

3 (a) (i) State the number of edges in a minimum spanning tree of a network with 10 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree of a network with n vertices. *(1 mark)*

(b) The following network has 10 vertices: A, B, ..., J. The numbers on each edge represent the distances, in miles, between pairs of vertices.



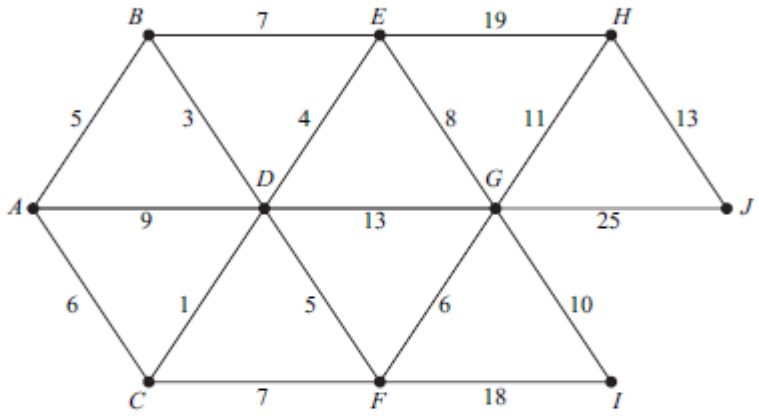
(i) Use Kruskal's algorithm to find the minimum spanning tree for the network. *(5 marks)*

(ii) State the length of your spanning tree. *(1 mark)*

(iii) Draw your spanning tree. *(2 marks)*

3 [**Figure 1**, printed on the insert, is provided for use in **part (b)** of this question.]

The diagram shows a network of roads. The number on each edge is the length, in kilometres, of the road.



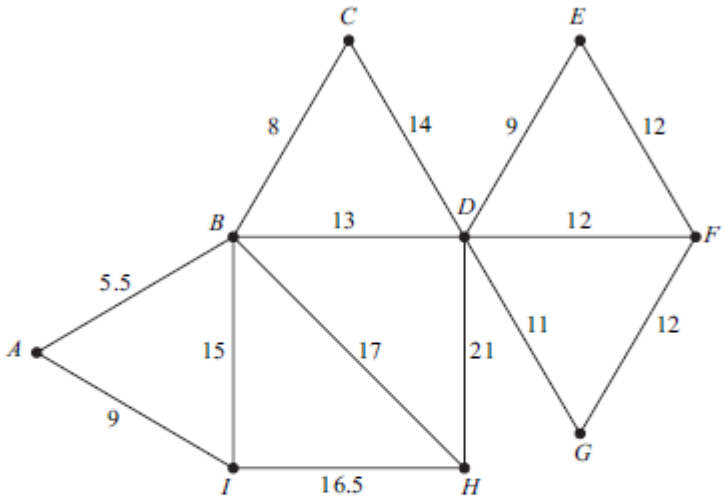
(a) (i) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(b) (i) Use Dijkstra's algorithm on **Figure 1** to find the shortest distance from A to J. *(6 marks)*

(ii) A new road, of length x km, is built connecting I to J. The minimum distance from A to J is reduced by using this new road. Find, and solve, an inequality for x. *(2 marks)*

1 The following network shows the lengths, in miles, of roads connecting nine villages.



(a) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

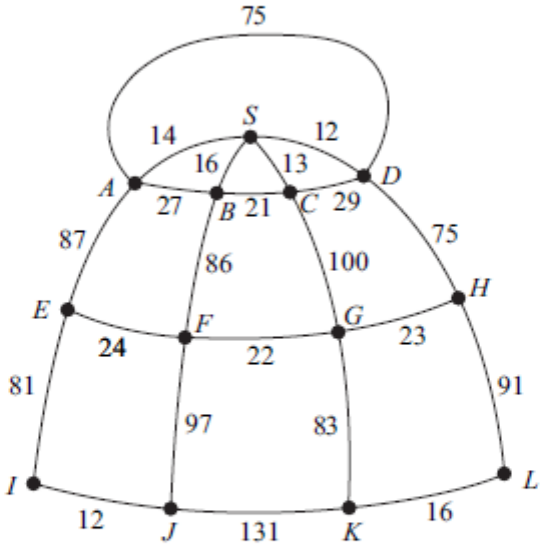
(b) Find the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

(d) State the number of other spanning trees that are of the same length as your answer in part (a). *(1 mark)*

4 The diagram shows the various ski-runs at a ski resort. There is a shop at S. The manager of the ski resort intends to install a floodlighting system by placing a floodlight at each of the 12 points A, B,...,L and at the shop at S.

The number on each edge represents the distance, in metres, between two points.



Total of all edges = 1135

(a) The manager wishes to use the minimum amount of cabling, which must be laid along the ski-runs, to connect the 12 points A, B,...,L and the shop at S.

(i) Starting from the shop, and showing your working at each stage, use Prim's algorithm to find the minimum amount of cabling needed to connect the shop and the 12 points. *(5 marks)*

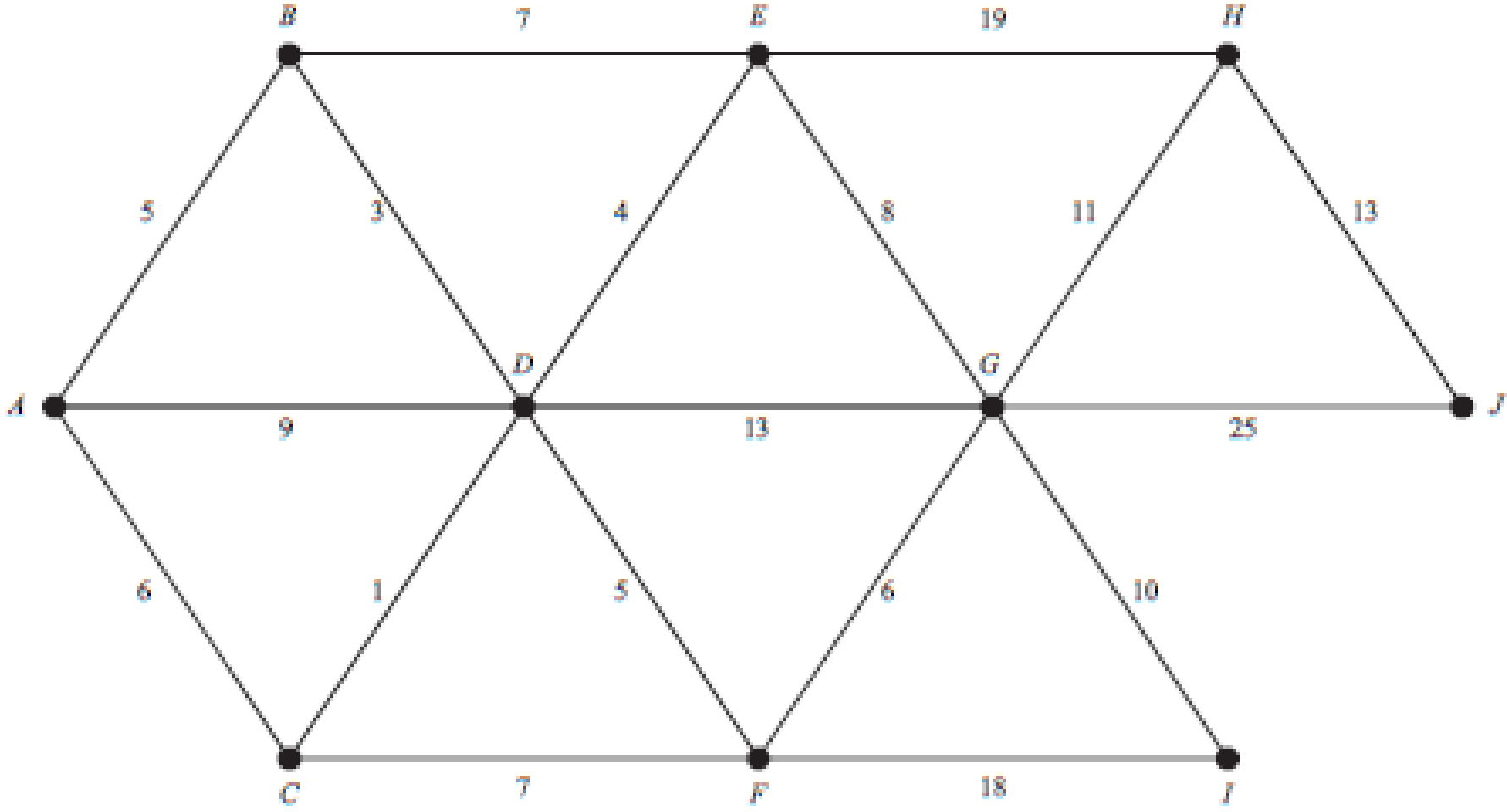
(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(3 marks)*

(iv) The manager used Kruskal's algorithm to find the same minimum spanning tree. Find the seventh and the eighth edges that the manager added to his spanning tree. *(2 marks)*

(b) At the end of each day a snow plough has to drive at least once along each edge shown in the diagram in preparation for the following day's skiing. The snow plough must start and finish at the point L.

Use the Chinese Postman algorithm to find the minimum distance that the snow plough must travel. *(6 marks)*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