

FP3 Numerical Methods for the Solution of First Order Differential Equations Answers

5(a)(i)	$y(1.1) = y(1) + 0.1[1 \ln 1 + 1/1]$ $= 1 + 0.1 = 1.1$	M1A1	3	
(ii)	$y(1.2) = y(1) + 2(0.1)[f(1.1, y(1.1))]$ $\dots = 1 + 2(0.1)[1.1 \ln 1.1 + (1.1)/1.1]$ $\dots = 1 + 0.2 \times 1.104841198 \dots$ $\dots = 1.22096824 = 1.221 \text{ to 3dp}$	M1A1	4	On answer to (a)(i) CAO
(b)(i)	$\text{IF is } e^{\int -\frac{1}{x} dx}$ $= e^{-\ln x}$ $= e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}$	M1	3	Condone $e^{\int \frac{1}{x} dx}$ for M mark AG (be convinced) (b)(i) Solutions using the printed answer must be convincing before any marks are awarded
(ii)	$\frac{d}{dx} \left(\frac{y}{x} \right) = \ln x$ $\frac{y}{x} = \int \ln x dx = x \ln x - \int x \left(\frac{1}{x} \right) dx$ $\frac{y}{x} = x \ln x - x + c$ $y(1) = 1 \Rightarrow 1 = \ln 1 - 1 + c$ $\Rightarrow c = 2 \Rightarrow y = x^2 \ln x - x^2 + 2x$	M1A1	6	Integration by parts for $x^k \ln x$ Condone missing c . Dependent on at least one of the two previous M marks OE eg $\frac{y}{x} = x \ln x - x + 2$
(iii)	$y(1.2) = 1.222543 \dots = 1.223 \text{ to 3dp}$	B1	1	
Total			17	

2(a)	$y_1 = 2 + 0.1 \times \left[\frac{1^2 + 2^2}{1 \times 2} \right]$ $= 2 + 0.1 \times 2.5 = 2.25$	M1 A1 A1	3	
(b)	$k_1 = 0.1 \times 2.5 = 0.25$ $k_2 = 0.1 \times f(1.1, 2.25)$ $\dots = 0.1 \times 2.53434\dots = 0.2534(34\dots)$ $y(1.1) = y(1) + \frac{1}{2}[0.25 + 0.253434\dots]$ $= 2.2517$ to 4dp	M1 A1✓ M1 A1✓ m1 A1✓	6	PI ft from (a) PI If answer not to 4dp withhold this mark
Total			9	

1(a)	$y(1.05) = 0.6 + 0.05 \times [\ln(1 + 1 + 0.6)]$ $= 0.6477(7557\dots) = 0.6478$ to 4dp	M1A1 A1	3	Condone >4 dp
(b)	$k_1 = 0.05 \times \ln(1 + 1 + 0.6) = 0.0477(75\dots)$ $k_2 = 0.05 \times f(1.05, 0.6477\dots)$ $\dots = 0.05 \times \ln(1 + 1.05^2 + 0.6477\dots)$ $\dots = 0.0505(85\dots)$ $y(1.05) = y(1) + \frac{1}{2}[k_1 + k_2]$ $= 0.6 + 0.5 \times 0.09836\dots$ $= 0.6492$ to 4dp	M1 A1F M1 A1F m1 A1F	6	PI ft candidate's evaluation in (a) PI Dep on previous two Ms and numerical values for k 's Must be 4 dp... ft one slip
Total			9	

2(a)	$y_1 = 2 + 0.1 \times \sqrt{1^2 + 2^2 + 3}$ $y(1.1) = 2 + 0.1 \times \sqrt{8}$ $y(1.1) = 2.28284\dots = 2.2828$ to 4dp	M1 A1 A1	3	
(b)	$k_1 = 0.1 \times \sqrt{8} = 0.2828$ $k_2 = 0.1 \times f(1.1, 2.2828\dots)$ $= 0.1 \times \sqrt{9.42137\dots} = 0.3069(425\dots)$ $y(1.1) = y(1) + \frac{1}{2}[0.28284\dots + 0.30694\dots]$ $2.29489\dots = 2.2949$ to 4dp	M1 A1ft M1 A1 m1 A1	6	PI PI
Total			9	