FP3 Polar Coordinates Answers

6(a)	$x^2 + y^2 - 12y + 36 = 36$	M1 M1		Use of $y = r\sin\theta$ ($x = r\cos\theta$ PI) Use of $x^2 + y^2 = r^2$
	$r^2 - 12r\sin\theta + 36 = 36$	m1		
	$\Rightarrow r = 12\sin\theta$	A1	4	**CSO AG**
(b)	Area $= \frac{1}{2}\int (2\sin\theta + 5)^2 \, d\theta$.	M1		Use of $\frac{1}{2}\int r^2 \, d\theta$.
	$.. = \frac{1}{2}\int_0^{2\pi} (4\sin^2\theta + 20\sin\theta + 25)d\theta$	B1 B1		Correct expn. of $(2\sin\theta+5)^2$ Correct limits
	$= \frac{1}{2}\int_0^{2\pi} (2(1 - \cos 2\theta) + 20\sin\theta + 25)d\theta$	M1		Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$.
	$= \frac{1}{2}[27\theta - \sin 2\theta - 20\cos\theta]_0^{2\pi}$	A1√		Correct integration ft wrong coeffs
	$= 27\pi$.	A1	6	CSO
(c)	At intersection $12\sin\theta = 2\sin\theta + 5$	M1		OE eg $r = 6(r - 5)$
	$\Rightarrow \sin\theta = \frac{5}{10}$	A1		OE eg $r = 6$
	Points $\left(6, \frac{\pi}{6}\right)$ and $\left(6, \frac{5\pi}{6}\right)$	A1		OE
	$OPMQ$ is a rhombus of side 6			Or two equilateral triangles of side 6
	Area $= 6 \times 6 \times \sin\frac{2\pi}{3}$ oe	M1 A1		Any valid complete method to find the area (or half area) of quadrilateral.
	$= 18\sqrt{3}$	A1	6	Accept unsimplified surd
	Total		**16**	

4(a)	Area $= \frac{1}{2}\int 36(1-\cos\theta)^2\,\mathrm{d}\theta$	M1		use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$
	$\ldots = \frac{1}{2}\int_0^{2\pi} 36(1-2\cos\theta+\cos^2\theta)\,\mathrm{d}\theta$	B1 B1		for correct explanation of $[6(1-\cos\theta)]^2$ for correct limits
	$=9\int_0^{2\pi} 2-4\cos\theta+(\cos 2\theta+1)\,\mathrm{d}\theta$	M1		Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$.
	$=\left[27\theta-36\sin\theta+\frac{9}{2}\sin 2\theta\right]_0^{2\pi}$	A1√		Correct integration; only ft if integrating $a + b\cos\theta + c\cos 2\theta$ with non-zero a, b, c.
	$=54\pi$	A1	6	CSO
(b)(i)	$x^2+y^2=9 \Rightarrow r^2=9$	B1		PI
	A & B: $3=6-6\cos\theta \Rightarrow \cos\theta=\frac{1}{2}$	M1		
	Pts of intersection $\left(3,\frac{\pi}{3}\right)$; $\left(3,\frac{5\pi}{3}\right)$	A1 A1√	4	OE (accept 'different' values of θ not in the given interval)
(ii)	Length $AB = 2\times r\sin\theta$	M1		
	………… $= 2\times 3\times\frac{\sqrt{3}}{2}=3\sqrt{3}$	A1	2	OE exact surd form
	Total		**12**	

2	$r-r\sin\theta=4$	M1		
	$r-y=4$	B1		$r\sin\theta = y$ stated or used
	$r=y+4$	A1		
	$x^2+y^2=(y+4)^2$	M1		$r^2=x^2+y^2$ used
	$x^2+y^2=y^2+8y+16$	A1F		ft one slip
	$y=\frac{x^2-16}{8}$	A1	6	
	Total		**6**	

7(a)	Area $= \frac{1}{2}\int(6+4\cos\theta)^2\,\mathrm{d}\theta$	M1		use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$
	$=\frac{1}{2}\left(\int_{-\pi}^{\pi}36+48\cos\theta+16\cos^2\theta\right)\mathrm{d}\theta$	B1 B1		for correct expansion of $[6+4\cos\theta)]^2$ for limits
	$=\left(\int_{-\pi}^{\pi}18+24\cos\theta+4(\cos 2\theta+1)\right)\mathrm{d}\theta$	M1		Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$
	$=[22\theta+24\sin\theta+2\sin 2\theta]_{-\pi}^{\pi}$	A1F		correct integration ft wrong coefficients
	$=44\pi$	A1	6	CSO
(b)	At P, $r=4$; At Q, $r=2$;	B1		PI
	P $\{x=\}$ $r\cos\theta=4\cos\frac{2\pi}{3}=-2$	M1		Attempt to use $r\cos\theta$
	Q $\{x=\}$ $r\cos\theta=2\cos\pi=-2$	A1		Both
	Since P and Q have same 'x', PQ is vertical so QP is parallel to the vertical line $\theta=\frac{\pi}{2}$	E1	4	

(c)(i)	$OP = 4$; $OS = 8$;	B1		
	Angle $POS = \frac{\pi}{3}$	B1		or $S\,(4, 4\sqrt{3})$ and $P\,(-2, 2\sqrt{3})$
	$PS^2 = 4^2 + 8^2 - 2\times 4\times 8\times \cos\frac{\pi}{3}$ oe	M1		Cosine rule used in triangle POS OE $PS^2 = (4+2)^2 + (4\sqrt{3} - 2\sqrt{3})^2$
	$PS = \sqrt{48}$ $\{= 4\sqrt{3}\}$	A1	4	
(ii)	Since $8^2 = 4^2 + \left(\sqrt{48}\right)^2$, $OS^2 = OP^2 + PS^2 \Rightarrow OPS$ is a right angle. (Converse of Pythagoras Theorem)	E1	1	Accept valid equivalents e.g. $PR = 2PQ = 2(2\sqrt{3}) = PS$. $\angle SRP = \angle RSP = \angle RPO = \frac{\pi}{6}$ $\Rightarrow$ OPS is a right angle
	Total		**15**	

4(a)	$(\cos\theta + \sin\theta)^2 = \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta$ $= 1 + \sin 2\theta$	B1	1	AG (be convinced)
(b)	$\left(x^2 + y^2\right)^3 = (x+y)^4$			
	$\left(r^2\right)^3 = (r\cos\theta + r\sin\theta)^4$	M2,1,0		[M1 for one of $x^2 + y^2 = r^2$ OE, $x = r\cos\theta$, $y = r\sin\theta$ used]
	$r^6 = r^4\,(\cos\,\theta + \sin\,\theta)^4$			
	$r^6 = r^4\,(1 + \sin\,2\theta)^2$	M1		Uses (a) OE at any stage
	$r^2 = (1 + \sin\,2\theta)^2$			
	$\Rightarrow r = (1 + \sin\,2\theta)$ $\{r \geq 0\}$	A1	4	CSO; AG
(c)(i)	$r = 0 \Rightarrow \sin\,2\theta = -1$			
	$2\theta = \sin^{-1}(-1)$; $= -\frac{\pi}{2}, \frac{3\pi}{2}$	M1		

	$\theta = -\frac{\pi}{4};\ \frac{3\pi}{4}$	A1A1ft	3	A1 for either
(ii)	Area $= \frac{1}{2}\int (1+\sin 2\theta)^2 \mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2 \mathrm{d}\theta$
	$= \frac{1}{2}\int (1 + 2\sin 2\theta + \sin^2 2\theta)\ \mathrm{d}\theta$	B1		Correct expansion of $(1+\sin 2\theta)^2$
	$= \frac{1}{2}\int \left(1 + 2\sin 2\theta + \frac{1}{2}(1-\cos 4\theta)\right) \mathrm{d}\theta$	M1		Attempt to write $\sin^2 2\theta$ in terms of $\cos 4\theta$
	$= \left[\frac{3}{4}\theta - \frac{1}{2}\cos 2\theta - \frac{1}{16}\sin 4\theta\right]$	A1ft		Correct integration ft wrong coefficients only
	$= \left[\frac{3}{4}\theta - \frac{1}{2}\cos 2\theta - \frac{1}{16}\sin 4\theta\right]_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}$			
	$= \left(\frac{9\pi}{16}\right) - \left(-\frac{3\pi}{16}\right)$	m1		Using c's values from (c)(i) as limits or the correct limits
	$= \frac{3\pi}{4}$	A1	6	CSO
	Total		**14**	