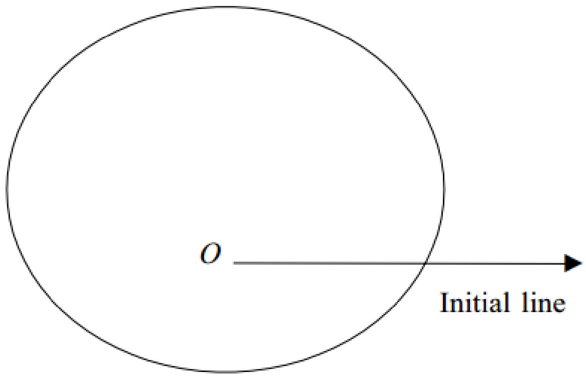
FP3 Polar Coordinates Questions

6 (a) A circle C_1 has cartesian equation $x^2 + (y-6)^2 = 36$. Show that the polar equation of C_1 is $r = 12\sin\theta$. *(4 marks)*

(b) A curve C_2 with polar equation $r = 2\sin\theta + 5$, $0 \leqslant \theta \leqslant 2\pi$ is shown in the diagram.

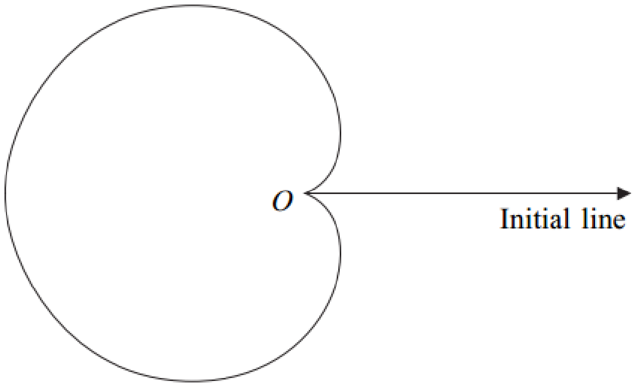


Calculate the area bounded by C_2. *(6 marks)*

(c) The circle C_1 intersects the curve C_2 at the points P and Q. Find, in surd form, the area of the quadrilateral $OPMQ$, where M is the centre of the circle and O is the pole. *(6 marks)*

4 The diagram shows the curve C with polar equation

$$r = 6(1 - \cos\theta), \qquad 0 \leqslant \theta < 2\pi$$



(a) Find the area of the region bounded by the curve C. *(6 marks)*

(b) The circle with cartesian equation $x^2 + y^2 = 9$ intersects the curve C at the points A and B.

(i) Find the polar coordinates of A and B. *(4 marks)*

(ii) Find, in surd form, the length of AB. *(2 marks)*

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2 A curve has polar equation $r(1 - \sin\theta) = 4$. Find its cartesian equation in the form $y = f(x)$. *(6 marks)*

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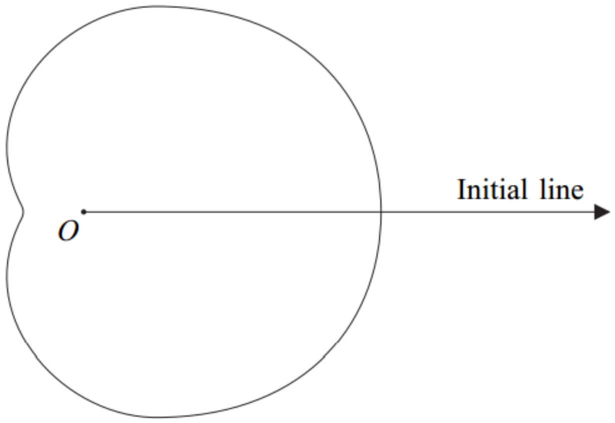
7 A curve C has polar equation

$$r = 6 + 4\cos\theta, \qquad -\pi \leqslant \theta \leqslant \pi$$

The diagram shows a sketch of the curve C, the pole O and the initial line.



(a) Calculate the area of the region bounded by the curve C. *(6 marks)*

(b) The point P is the point on the curve C for which $\theta = \frac{2\pi}{3}$.

The point Q is the point on C for which $\theta = \pi$.

Show that QP is parallel to the line $\theta = \frac{\pi}{2}$. *(4 marks)*

(c) The line PQ intersects the curve C again at a point R.

The line RO intersects C again at a point S.

(i) Find, in surd form, the length of PS. *(4 marks)*

(ii) Show that the angle OPS is a right angle. *(1 mark)*

4 (a) Show that $(\cos\theta + \sin\theta)^2 = 1 + \sin 2\theta$. *(1 mark)*

(b) A curve has cartesian equation

$$(x^2 + y^2)^3 = (x + y)^4$$

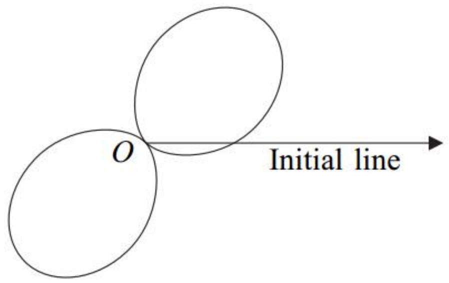
Given that $r \geqslant 0$, show that the polar equation of the curve is

$$r = 1 + \sin 2\theta$$ *(4 marks)*

(c) The curve with polar equation

$$r = 1 + \sin 2\theta, \quad -\pi \leqslant \theta \leqslant \pi$$

is shown in the diagram.



(i) Find the two values of θ for which $r = 0$. *(3 marks)*

(ii) Find the area of one of the loops. *(6 marks)*