FP3 Second Order Differential Equation Questions

1 (a) Find the roots of the equation $m^2 + 2m + 2 = 0$ in the form $a + ib$. *(2 marks)*

(b) (i) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 4x$$

(6 marks)

(ii) Hence express y in terms of x, given that $y = 1$ and $\frac{dy}{dx} = 2$ when $x = 0$.

(4 marks)

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1 It is given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 8x - 10 - 10\cos 2x$$

(a) Show that $y = 2x + \sin 2x$ is a particular integral of the given differential equation. *(3 marks)*

(b) Find the general solution of the differential equation. *(4 marks)*

(c) Hence express y in terms of x, given that $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$. *(4 marks)*

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6 (a) Show that the substitution

$$u = \frac{dy}{dx} + 2y$$

transforms the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$$

into

$$\frac{du}{dx} + 2u = e^{-2x}$$ *(4 marks)*

(b) By using an integrating factor, or otherwise, find the general solution of

$$\frac{du}{dx} + 2u = e^{-2x}$$

giving your answer in the form $u = f(x)$. *(5 marks)*

(c) Hence find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$$

giving your answer in the form $y = g(x)$. *(5 marks)*

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5 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 6 + 5\sin x$$ *(12 marks)*

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1 (a) Find the value of the constant k for which kx^2e^{5x} is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 6e^{5x}$$ *(6 marks)*

(b) Hence find the general solution of this differential equation. *(4 marks)*

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5 (a) A differential equation is given by

$$(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} = x^2 + 1$$

Show that the substitution

$$u = \frac{dy}{dx} + x$$

transforms this differential equation into

$$\frac{du}{dx} = \frac{2xu}{x^2 - 1}$$ *(4 marks)*

(b) Find the general solution of

$$\frac{du}{dx} = \frac{2xu}{x^2 - 1}$$

giving your answer in the form $u = f(x)$. *(5 marks)*

(c) Hence find the general solution of the differential equation

$$(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} = x^2 + 1$$

giving your answer in the form $y = g(x)$. *(3 marks)*