Mechanics 2 Calculus in Kinematics

3(a)(i)	$a = 2 + 12e^{-t}$	M1		Differentiating, with at least one term correct.
		A1	2	Correct velocity
(ii)	$2 < a \le 14$	B1		For 2
		B1		For 14
		B1	3	Correct inequalities
(b)	$s = t^2 + 12e^{-t} + c$	M1		Integrating, with at least one term correct
	$s = 0, t = 0 \Rightarrow c = -12$	A1		Correct expression with or without c
		dM1		Finding c.
	$s = t^2 + 12e^{-t} - 12$	A1	4	Correct final expression
	Total		**9**	

5(a)	$\mathbf{F} = 12\cos t\mathbf{i} - 30\sin t\mathbf{j}$	M1		Use of $\mathbf{F} = m\mathbf{a}$
	$\mathbf{F}(0) = 6\times 2\mathbf{i}$ so $F = 12$ N	A1		Correct $\mathbf{F}$
		A1	3	Correct magnitude
(b)	$\mathbf{v} = \int 2\cos t\,dt\mathbf{i} + \int -5\sin t\,dt\mathbf{j}$	M1		Integrating
	$= (2\sin t + c_1)\mathbf{i} + (5\cos t + c_2)\mathbf{j}$	A1		Correct **i** component
		A1		Correct **j** component Both of above with or without constants
	$\mathbf{v}(0) = 2\mathbf{i} + 10\mathbf{j} \Rightarrow c_1 = 2, c_2 = 5$	dM1		Finding constants of integration
	$\mathbf{v} = (2\sin t + 2)\mathbf{i} + (5\cos t + 5)\mathbf{j}$	A1	5	Correct final answer
	Total		**8**	

1(a)	$\mathbf{v} = (6t^2 - 2t)\mathbf{i} + (1 - 12t^2)\mathbf{j}$	M1		differentiating both components
		A1		one component correct
		A1	3	second component correct
(b)(i)	$\mathbf{v}\left(\frac{1}{3}\right) = \left(\frac{6}{9} - \frac{2}{3}\right)\mathbf{i} + \left(1 - \frac{12}{9}\right)\mathbf{j} = -\frac{1}{3}\mathbf{j}$	M1		substituting the value for t into their **v**
		A1	2	correct velocity
(ii)	Travelling due south	A1ft	1	correct description (Follow through from $\mathbf{v} = \pm k\mathbf{j}$)
(c)	$\mathbf{a} = (12t - 2)\mathbf{i} - 24t\mathbf{j}$	M1		differentiating their velocity
	$\mathbf{a}(4) = 46\mathbf{i} - 96\mathbf{j}$	A1		correct acceleration at time t
		A1	3	correct acceleration at $t = 4$
(d)	$\mathbf{F} = 6(46\mathbf{i} - 96\mathbf{j}) = 276\mathbf{i} - 576\mathbf{j}$	M1		apply Newton's second law correctly
	$F = \sqrt{276^2 + 576^2} = 639$ N	M1		finding magnitude
		A1	3	correct magnitude
	or $a = \sqrt{46^2 + 96^2} = 106.45$ $F = 6 \times 106.45 = 639$ N			
	Total		**12**	

5(a)	$F = 800 + \frac{1200}{20}t = 800 + 60t$	M1		finding the gradient of the line
	$1200a = 800 + 60t$	A1		correct gradient
	$a = \frac{800}{1200} + \frac{60}{1200}t = \frac{2}{3} + \frac{t}{20}$	B1		correct intercept
		dM1		using Newton's second law on two terms
	AG	A1	5	correct result from correct working
(b)	$v = \int \frac{2}{3} + \frac{t}{20} \mathrm{d}t = \frac{2t}{3} + \frac{t^2}{40} + c$	M1		integrating
	$v = 0, t = 0 \Rightarrow c = 0$	A1		correct integral with or without c
	$v = \frac{2t}{3} + \frac{t^2}{40}$	A1	3	showing $c = 0$
(c)	$s = \int_0^{20} \frac{2t}{3} + \frac{t^2}{40} \mathrm{d}t$	M1		integrating
		A1		correct integral, with or without c.
	$= \left[\frac{t^2}{3} + \frac{t^3}{120}\right]_0^{20}$	dM1		use of both limits or finding c
	$= 200$ m	A1	4	correct distance
(d)	The $\frac{2t}{3}$ term would change, because only	B1		correct term
	the constant term in the force would change. When integrated this becomes the t term in the velocity.	B1	2	correct explanation
	Total		**14**	

5(a)(i)	$t = 0,\ \mathbf{r} = 2\mathbf{i} + 10\mathbf{k}$	B1	1	
(ii)	$t = 2\pi,\ \mathbf{r} = 2\mathbf{i} + 7.49\mathbf{k}$	B1	1	Or $\mathbf{r} = 2\mathrm{i} + (10 - 0.8\pi)\mathbf{k}$ accept $7.5\mathbf{k}$
(iii)	$t = 2\pi, \quad t = 4\pi$	B1		
		B1	2	
(b)	$\mathbf{v} = -2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j} - 0.4\mathbf{k}$	M1		Differentiation
		A1		Trig
		A1	3	**k**
(c)	$\mathbf{a} = -2\cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$	M1A1		
	$\mathbf{F} = -50\cos t\,\mathbf{i} - 50\sin t\,\mathbf{j}$	M1		
	$\lvert\mathbf{F}\rvert = \sqrt{50^2\cos^2 t + 50^2\sin^2 t}$	M1		No unit vectors
	$\lvert\mathbf{F}\rvert = 50(\mathrm{N})$	A1	5	
	Total		**12**	

3(a)	Using $F = ma$:			
	$2400\mathbf{i} - 4800t\mathbf{j} = 800\mathbf{a}$	M1		
	$\mathbf{a} = 3\mathbf{i} - 6t\mathbf{j}$	A1	2	
(b)	$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t$	M1		
	$= 3t\mathbf{i} - 3t^2\mathbf{j} + \mathbf{c}$	A1		Condone no '+ **c**'
	When $t = 0$, $\mathbf{v} = 6\mathbf{i} + 30\mathbf{j}$			
	$\therefore\ \mathbf{c} = 6\mathbf{i} + 30\mathbf{j}$	M1		Needs '+ **c**' above
	$\therefore\ \mathbf{v} = (3t + 6)\mathbf{i} + (30 - 3t^2)\mathbf{j}$	A1	4	AG
(c)	$\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t$	M1		
	$= (\frac{3}{2}t^2 + 6t)\mathbf{i} + (30t - t^3)\mathbf{j} + \mathbf{d}$	A1,A1		A1 **i** term, A1 **j** term; condone no '+ **d**'
	When $t = 0$, $\mathbf{r} = 2\mathbf{i} + 5\mathbf{j}$			
	$\therefore\ \mathbf{d} = 2\mathbf{i} + 5\mathbf{j}$	M1		
	$\therefore\ \mathbf{r} = (\frac{3}{2}t^2 + 6t + 2)\mathbf{i} + (30t - t^3 + 5)\mathbf{j}$	A1	5	
	Total		**11**	