Mechanics 2 Calculus in Kinematics

3 A particle moves in a straight line and at time t has velocity v, where

$$v = 2t - 12e^{-t}, \quad t \geqslant 0$$

(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(ii) State the range of values of the acceleration of the particle. *(3 marks)*

(b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

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5 A particle moves such that at time t seconds its acceleration is given by

$$(2\cos t\mathbf{i} - 5\sin t\mathbf{j})\,\mathrm{m\,s^{-2}}$$

(a) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 0$. *(3 marks)*

(b) When $t = 0$, the velocity of the particle is $(2\mathbf{i} + 10\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Find an expression for the velocity of the particle at time t. *(5 marks)*

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1 A particle moves in a horizontal plane, in which the unit vectors **i** and **j** are directed east and north respectively. At time t seconds, its position vector, **r** metres, is given by

$$\mathbf{r} = (2t^3 - t^2 + 6)\mathbf{i} + (8 - 4t^3 + t)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(3 marks)*

(b) (i) Find the velocity of the particle when $t = \frac{1}{3}$. *(2 marks)*

(ii) State the direction in which the particle is travelling at this time. *(1 mark)*

(c) Find the acceleration of the particle when $t = 4$. *(3 marks)*

(d) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 4$. *(3 marks)*

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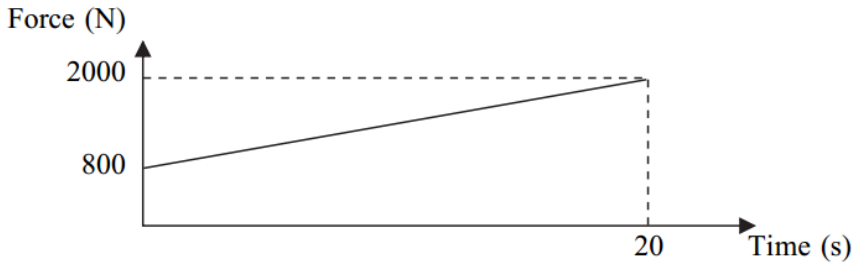
5 The graph shows a model for the resultant horizontal force on a car, which varies as it accelerates from rest for 20 seconds. The mass of the car is 1200 kg.



(a) The acceleration of the car at time t seconds is a m s^{-2}. Show that

$$a = \frac{2}{3} + \frac{t}{20}, \quad \text{for} \quad 0 \leqslant t \leqslant 20$$

(5 marks)

(b) Find an expression for the velocity of the car at time t. *(3 marks)*

(c) Find the distance travelled by the car in the 20 seconds. *(4 marks)*

(d) An alternative model assumes that the resultant force increases uniformly from 900 to 2100 newtons during the 20 seconds. Which term in your expression for the velocity would change as a result of this modification? Explain why. *(2 marks)*

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5 Tom is on a fairground ride.

Tom's position vector, **r** metres, at time t seconds is given by

$$\mathbf{r} = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + (10 - 0.4t)\mathbf{k}$$

The perpendicular unit vectors **i** and **j** are in the horizontal plane and the unit vector **k** is directed vertically upwards.

(a) (i) Find Tom's position vector when $t = 0$. *(1 mark)*

(ii) Find Tom's position vector when $t = 2\pi$. *(1 mark)*

(iii) Write down the first **two** values of t for which Tom is directly below his starting point. *(2 marks)*

(b) Find an expression for Tom's velocity at time t. *(3 marks)*

(c) Tom has mass 25 kg.

Show that the resultant force acting on Tom during the motion has constant magnitude. State the magnitude of the resultant force. *(5 marks)*

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3 A particle has mass 800 kg. A single force of $(2400\,\mathbf{i} - 4800t\,\mathbf{j})$ newtons acts on the particle at time t seconds. No other forces act on the particle.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) At time $t = 0$, the velocity of the particle is $(6\mathbf{i} + 30\mathbf{j})\,\mathrm{m\,s^{-1}}$. The velocity of the particle at time t is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

Show that

$$\mathbf{v} = (6 + 3t)\,\mathbf{i} + (30 - 3t^2)\,\mathbf{j}$$ *(4 marks)*

(c) Initially, the particle is at the point with position vector $(2\mathbf{i} + 5\mathbf{j})\,\mathrm{m}$.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*