Mechanics 2 Differential Equations

6(a)	The boat is not affected by the movement of the water.	B1		First assumption
	The resistance force will directly oppose the motion of the boat and be the only force that needs to be considered.	B1	2	second assumption
(b)	The boat is a particle. / There is no wind. / No air resistance.	B1	1	Appropriate assumption
(c)(i)	$80\frac{dv}{dt} = -20v$	M1		Use of Newton's second law, $\frac{dv}{dt}$ and $20v$.
	$\frac{dv}{dt} = -\frac{v}{4}$	A1	2	Correct result
(ii)	$\int \frac{1}{v} dv = -\int \frac{1}{4} dt$	M1		Sep. of variables and forming integrals
	$\ln v = -\frac{t}{4} + c$	dM1 A1		Integrating to get an ln v term Correct integrals with or without c
	$v = Ae^{-\frac{t}{4}}$ $v = 12, t = 0 \Rightarrow A = 12$	dM1		Finding A or c
	$v = 12e^{-\frac{t}{4}}$	A1	5	Correct final result
	Total		**10**	

7(a)	$20\frac{dv}{dt} = -10\sqrt{v}$	M1		applying Newton's second law with $\frac{dv}{dt}$
	$\frac{dv}{dt} = -\frac{\sqrt{v}}{2}$	A1		correct differential equation
	$\int \frac{1}{\sqrt{v}} dv = \int -\frac{1}{2} dt$ AG	dM1		separating variables
	$2\sqrt{v} = -\frac{t}{2} + c$	dM1		integrating
	$t = 0, v = 25 \Rightarrow c = 10$	A1 dM1		correct integrals with or without c finding the constant of integration
	$v = \left(\frac{20 - t}{4}\right)^2$	A1	7	correct final result from correct working
(b)	$t = 20$	B1	1	correct time
	Total		**8**	

7(a)	Max speed ≡ zero acceleration used	M1		Implied
	$\frac{72000}{60}$	M1		
	$\frac{72000}{60} = k \times 60$			
	$k = 20$	A1	3	
(b)(i)	$20v = -500\frac{dv}{dt}$	M1		see $\frac{dv}{dt}$, ±
	$\frac{dv}{dt} = -\frac{v}{25}$	A1	2	
(ii)	$25\int \frac{dv}{v} = -\int dt$	M1 A1		M1 separating variables
	$[25\ln v]_{20}^{10} = -[t]_0^t$	A1		**Alternative** $25\ln v = -t\ (+c)$ A1
	$25\ln 10 - 25\ln 20 = -t$	m1 A1		$t = 0,\ v = 20,\ c = 25\ln 20$ m1 $t = t,\ v = 10,$ $25\ln 10 = -t + 25\ln 20$ A1
	$t = 25\ln 2$ or 17.3 or $-25\ln\frac{1}{2}$	A1	6	$t = 25\ln 2$ or 17.3 A1
	Total		**11**	

7(a)	Using $F = ma$:			
	$-\lambda mv = ma = m\frac{dv}{dt}$	M1		Condone no '–'
	$\therefore \frac{dv}{dt} = -\lambda v$	A1	2	AG Note: no use of $m \Rightarrow$ no marks in (a)
(b)	$\int \frac{dv}{v} = -\lambda\int dt$	M1		
	$\ln v = -\lambda t + c$ $v = Ce^{-\lambda t}$	A1		Needs '+ c'
	When $t = 0,\ v = U \Rightarrow C = U$	M1		Needs correct working
	$v = Ue^{-\lambda t}$	A1	4	AG
	Total		**6**	
