Mechanics 2 Differential Equations

6 A student is modelling the motion of a small boat as it moves on a lake. When the speed of the boat is $12\,\text{m s}^{-1}$, the engine is switched off. At time t seconds later, it has a velocity of $v\,\text{m s}^{-1}$ and experiences a resistance force of magnitude $20v$ newtons. The mass of the boat is 80 kg.

To set up a simple model for the motion of the boat, the student assumes that the water in the lake is still and that the boat travels in a straight line.

(a) Explain how these two assumptions allow the student to create a simple model. *(2 marks)*

(b) State one other assumption that the student should make. *(1 mark)*

(c) (i) Express $\frac{dv}{dt}$ in terms of v. *(2 marks)*

(ii) Find an expression for v in terms of t. *(5 marks)*

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7 A particle of mass 20 kg moves along a straight horizontal line. At time t seconds the velocity of the particle is $v\,\text{m s}^{-1}$. A resistance force of magnitude $10\sqrt{v}$ newtons acts on the particle while it is moving. At time $t = 0$ the velocity of the particle is $25\,\text{m s}^{-1}$.

(a) Show that, at time t

$$v = \left(\frac{20 - t}{4}\right)^2$$

(7 marks)

(b) State the value of t when the particle comes to rest. *(1 mark)*

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7 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V\,\text{m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is $60\,\text{m s}^{-1}$.

Show that $k = 20$. *(3 marks)*

(b) When the motorcycle is travelling at $20\,\text{m s}^{-1}$, the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v\,\text{m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

The mass of the motorcycle and its rider is 500 kg.

(i) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -\dfrac{v}{25}$. *(2 marks)*

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from $20\,\text{m s}^{-1}$ to $10\,\text{m s}^{-1}$. *(6 marks)*

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7 A stone of mass m is moving along the smooth horizontal floor of a tank which is filled with a viscous liquid. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude λmv, where λ is a constant.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\lambda v$$ *(2 marks)*

(b) The initial speed of the stone is U.

Show that

$$v = U\mathrm{e}^{-\lambda t}$$ *(4 marks)*
