# Stats 2 Continuous Random Variable Answers

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|---|---|---|---|---|
| **4(a)(i)** | Area $= k(b-a) = 1$ | | | |
| | $\Rightarrow \quad k = \dfrac{1}{b-a}$ | E1 | 1 | AG |
| **(ii)** | $\mathrm{E}(X) = \int_a^b kx \, \mathrm{d}x$ | M1 | | |
| | $= \left(\dfrac{kx^2}{2}\right)\Big\vert_a^b$ | A1 | | |
| | $= \dfrac{1}{2}k\left(b^2 - a^2\right)$ | | | |
| | $= \dfrac{1}{2} \times \dfrac{1}{(b-a)} \times (b-a)(a+b)$ | M1A1 | | (factors shown) |
| | $= \dfrac{1}{2}(a+b)$ | | 4 | AG |
| **(b)(i)** | $\mu = 1$ | B1 | 1 | |
| **(ii)** | $\sigma^2 = \mathrm{Var}(X) = \dfrac{1}{12}(b-a)^2$ | | | |
| | $= \dfrac{1}{12} \times 6^2$ | M1 | | |
| | $= 3$ | | | |
| | $\therefore \sigma = \sqrt{3}$ | A1 | 2 | 1.7321 |
| **(iii)** | $\mathrm{P}\left(X < \dfrac{2-\mu}{\sigma}\right) = \mathrm{P}\left(X < \dfrac{1}{\sqrt{3}}\right)$ | M1√ | | (on their $\mu$ and $\sigma$) |
| | $= \dfrac{1}{6} \times 2.577$ | M1√ | | |
| | $= 0.430$ | A1 | 3 | cao |
| | **Total** | | **11** | |

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| **7(a)** | $E(T)=\int_0^1 t\,f(t)\,dt$ | | | |
| | $=\int_0^1 4t^2\left(1-t^2\right)dt$ | M1 | | |
| | $=\left(\frac{4t^3}{3}-\frac{4t^5}{5}\right)\Big\vert_0^1$ | A1 | | |
| | $=\frac{4}{3}-\frac{4}{5}$ | A1 | | |
| | $=\frac{8}{15}$ | | 3 | AG |
| **(b)(i)** | $F(t)=P(T\le t)=\int_0^t f(t)\,dt$ | | | |
| | $=\int_0^t 4t\left(1-t^2\right)dt$ | M1 | | |
| | $=\left(2t^2-t^4\right)\Big\vert_0^t$ | | | |
| | $=2t^2-t^4$ | A1 | 2 | |
| **(ii)** | $P(\mu<T<m)=F(m)-F(\mu)$ | M1 | | |
| | $\Downarrow$ | | | |
| | $F(m)=0.5$ | B1 | | |
| | $F(\mu)=F\left(\frac{8}{15}\right)=0.4880$ | B1 | | |
| | $\therefore\ P(\mu<T<m)=0.5-0.4880$ | M1√ | | 0.5 – their $F(\mu)$ |
| | $=0.012$ | A1 | 5 | |
| | **Total** | | **10** | |

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| 5(a)(i) | $E(X)=\frac{1}{2}b$ | B1 | 1 | |
| (ii) | $E(X^2)=\int_0^b \frac{1}{b}x^2 dx$ | M1 | | |
| | $=\frac{1}{b}\left[\frac{x^3}{3}\right]_0^b$ | A1 | | For correct integration |
| | $=\frac{1}{b}\left(\frac{b^3}{3}\right)$ | | | |
| | $=\frac{1}{3}b^2$ | A1 | | OE |
| | $\text{Var}(X)=\frac{1}{3}b^2-\left(\frac{b}{2}\right)^2$ | m1 | | Depending on using integration to get $E(X^2)$ |
| | $=\frac{1}{3}b^2-\frac{1}{4}b^2$ | | | |
| | $=\frac{1}{12}b^2$ | A1 | 5 | AG |
| (b) | $P(\lvert T\rvert>0.02)=1-P(-0.02<T<0.02)$ | M1 | | |
| | $=1-0.04\times5$ | M1 | | |
| | $=0.8$ | A1 | 3 | |
| | **Total** | | **9** | |

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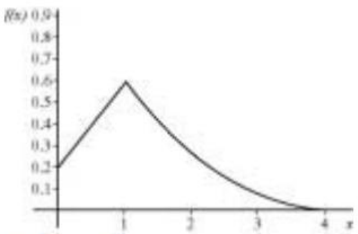
|---|---|---|---|---|
| 7(a) |  | B2 | 2 | B1 for line segment (0,0.2) to (1,0.6)<br>B1 for correctly shaped curve (1,0.6) to (4,0) |
| (b)(i) | for $0\le x\le1$ | | | |
| | $F(x)=\int_0^x \frac{1}{5}(2x+1)dx$ | M1 | | Ignore limits |
| | $=\left[\frac{1}{5}(x^2+x)\right]_0^x$ | A1 | | Ignore limits |
| | $=\frac{1}{5}x(x+1)$ | A1 | 3 | |
| (ii) | $P(X\le1)=F(1)$ | | | |
| | $=\frac{2}{5}$ | B1 | 1 | |

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| **(iii)** | $\mathrm{P}(X \ge x) = \frac{17}{20} \quad \Rightarrow \quad \mathrm{F}(x) = \frac{3}{20}$ | M1 | | |
| | $\frac{1}{5}x(x+1) = \frac{3}{20}$ | m1 | | |
| | $x(x+1) = \frac{3}{4}$ | | | |
| | $x^2 + x - \frac{3}{4} = 0$ | A1 | | |
| | $\left(x - \frac{1}{2}\right)\left(x + \frac{3}{2}\right) = 0$ | m1 | | Any valid method attempted |
| | $x = \frac{1}{2}$ | A1 | 5 | CAO |
| **(iv)** | Since $\mathrm{F}(1) = 0.4$, $q$ lies in $0 \le r \le 1$ | | | |
| | $\mathrm{F}(q) = \frac{1}{5}\left(q^2 + q\right) = 0.25$ | M1 | | |
| | $\Rightarrow \quad q^2 + q = 1.25$ | | | |
| | $q^2 + q - 1.25 = 0$ | A1 | | |
| | $\Rightarrow \quad q = \frac{-1 \pm \sqrt{1 - 4 \times (-1.25)}}{2}$ | m1 | | |
| | $q = \frac{1}{2}\left(\sqrt{6} - 1\right) \quad (q > 0)$ | A1 | 4 | AWFW (0.724 to 0.725) |
| | **Total** | | **15** | |

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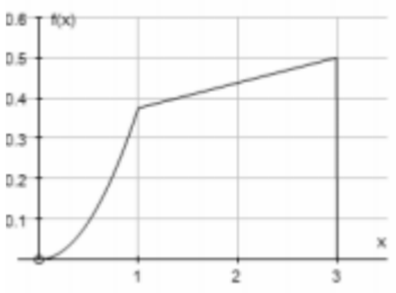
|---|---|---|---|---|
| **6(a)** |  | B1<br>B1<br>B1 | 3 | for curve<br>for line<br>for axes |
| **(b)** | $\mathrm{P}(T \ge 1) = \frac{1}{2} \times \frac{7}{8} \times 2 = \frac{7}{8}$ | M1A1 | 2 | OE |

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| **6(c)(i)** | For $1 \le t \le 3$ | | | |
| | $\int_1^t \frac{1}{16}(t+5)\,dt = \left[\frac{1}{32}t^2 + \frac{5}{16}t\right]_1^t$ | M1A1 | | |
| | $F(1) = \frac{1}{8}$ | B1 | | |
| | $F(t) = \frac{1}{8} + \frac{1}{32}t^2 + \frac{5}{16}t - \frac{11}{32}$ | M1 | | Use of: $F(t) = F(1) + \int_1^t \frac{1}{16}(t+5)\,dt$ |
| | $F(t) = \frac{1}{32}\left(t^2 + 10t - 7\right)$ | A1 | 5 | **AG** |
| | **Alternative:** | | | |
| | $\int \frac{1}{16}(t+5)\,dt$ | | | |
| | $= \frac{1}{16}\left(\frac{1}{2}t^2 + 5t + c\right)$ | (M1)<br>(A1) | | |
| | $F(1) = \frac{1}{8}$ | (B1) | | |
| | $\Rightarrow c = -3.5$ | (M1) | | |
| | $F(t) = \frac{1}{32}\left(t^2 + 10t - 7\right)$ | (A1) | | |
| **(ii)** | $\frac{1}{32}\left(m^2 + 10m - 7\right) = 0.5$ | M1 | | |
| | $m^2 + 10m - 23 = 0$ | A1 | | |
| | $m = \frac{-10 \pm \sqrt{192}}{2} = -5 \pm \sqrt{48}$<br>$= -5 \pm 4\sqrt{3}$ | m1 | | (or any valid method) |
| | $(m > 0)$<br>$m = 4\sqrt{3} - 5 = 1.93$ | A1 | 4 | (1.9282) |
| | **Total** | | **14** | |

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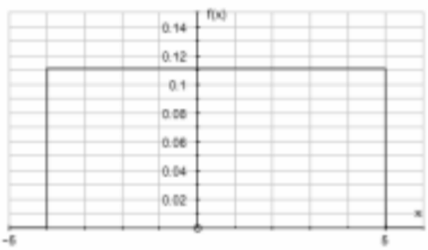
| **8(a)** | $f(x) = \begin{cases} \frac{1}{9} & -4 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$ | M1<br>A1 | 2 | |
| **(b)** | (graph below) | B1 | | horizontal line from –4 to 5 |
| | | B1 | 2 | for drawn at $\frac{1}{9}$ |

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|---|---|---|---|---|
| **(c)** | $P(X > 2) = \frac{1}{9} \times 3$ | M1 | | $F(5) - F(2)$<br>$= 1 - \frac{2}{3}$ |
| | $= \frac{1}{3}$ | A1 | 2 | $= \frac{1}{3}$ |

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| **(d)** | Mean $=\frac{1}{2}$ | B1 | | |
| | Variance $=\frac{1}{12}\times 81$ | | | |
| | $=6.75$ | B1 | 2 | |
| | **Total** | | **8** | |

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|---|---|---|---|---|
| **4(a)** | For a Rectangular Distribution | | | |
| | $f(x)=\begin{cases}\frac{1}{b-a} & a\le x\le b\\ 0 & \text{otherwise}\end{cases}$ | | | |
| | $(-0.05, 0.05)\Rightarrow$ | B1 | | (explain error $\pm 0.05$) |
| | $\frac{1}{b-a}=\frac{1}{0.05-(-0.05)}=\frac{1}{0.1}=10$ | M1<br>A1 | 3 | |
| | (Area $=10\times 0.1=1$) | | | |
| **(b)** | $P(-0.01<X<0.02)=0.03\times 10=0.3$ | M1<br>A1 | 2 | |
| **(c)** | Mean $=0$ | B1 | | CAO |
| | Standard deviation $=0.0289$ | B1 | 2 | $\frac{1}{20\sqrt{3}}$ OE |
| | **Total** | | **7** | |

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| **6(a)(i)** | $E\left(\frac{1}{X}\right)=\int_0^1 \frac{1}{x}3x^2\,dx=\int_0^1 3x\,dx$ | M1 | | |
| | $=\left[\frac{3x^2}{2}\right]_0^1=1.5$ | A1<br>A1 | 3 | CAO |
| **(ii)** | $E\left(\frac{1}{X^2}\right)=\int_0^1 \frac{1}{x^2}3x^2\,dx=\int_0^1 3\,dx$ | M1 | | |
| | $=[3x]_0^1=3.0$ | A1 | | |
| | $\text{Var}\left(\frac{1}{X}\right)=3.0-(1.5)^2$ | m1 | | dependent on previous M1 |
| | $=0.75$ | A1√ | 4 | [on their (i)] and Var > 0 |
| **(b)** | $E\left(\frac{5+2X}{X}\right)=E\left(\frac{5}{X}+2\right)$ | M1 | | |
| | $=5E\left(\frac{1}{X}\right)+2$ | M1 | | |
| | $=5\times 1.5+2$<br>$=9.5$ | A1 | | CAO |
| | $\text{Var}\left(\frac{5+2X}{X}\right)=\text{Var}\left(\frac{5}{X}+2\right)$ | | | |
| | $=25\times\text{Var}\left(\frac{1}{X}\right)$ | M1 | | |
| | $=25\times 0.75$<br>$=18.75$ | A1 | 5 | CAO |
| | **Total** | | **12** | |