

6.  $f(x) = x - [x], x \geq 0$

where  $[x]$  is the largest integer  $\leq x$ .

For example,  $f(3.7) = 3.7 - 3 = 0.7$ ;  $f(3) = 3 - 3 = 0$ .

(a) Sketch the graph of  $y = f(x)$  for  $0 \leq x < 4$ . (3)

(b) Find the value of  $p$  for which  $\int_2^p f(x) dx = 0.18$ . (3)

Given that

$$g(x) = \frac{1}{1+kx}, \quad x \geq 0, \quad k > 0,$$

and that  $x_0 = \frac{1}{2}$  is a root of the equation  $f(x) = g(x)$ ,

(c) find the value of  $k$ . (2)

(d) Add a sketch of the graph of  $y = g(x)$  to your answer to part (a). (1)

The root of  $f(x) = g(x)$  in the interval  $n < x < n + 1$  is  $x_n$ , where  $n$  is an integer.

(e) Prove that

$$2x_n^2 - (2n - 1)x_n - (n + 1) = 0. \quad (4)$$

(f) Find the smallest value of  $n$  for which  $x_n - n < 0.05$ . (4)