7. Triangle ABC, with $BC = a$, $AC = b$ and $AB = c$ is inscribed in a circle. Given that AB is a diameter of the circle and that a^2, b^2 and c^2 are three consecutive terms of an arithmetic progression (arithmetic series),

(a) express b and c in terms of a,

(4)

(b) verify that cot A, cot B and cot C are consecutive terms of an arithmetic progression.

(3)

In an acute-angled triangle PQR the sides QR, PR and PQ have lengths p, q and r respectively.

(c) Prove that

$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}.$$

(3)

Given now that triangle PQR is such that p^2, q^2 and r^2 are three consecutive terms of an arithmetic progression,

(d) use the cosine rule to prove that $\dfrac{2\cos Q}{q} = \dfrac{\cos P}{p} + \dfrac{\cos R}{r}$.

(6)

(e) Using the results given in parts (c) and (d), prove that cot P, cot Q and cot R are consecutive terms in an arithmetic progression.

(3)