4. A rectangle $ABCD$ is drawn so that A and B lie on the x-axis, and C and D lie on the curve with equation $y = \cos x, -\frac{\pi}{2} < x < \frac{\pi}{2}$. The point A has coordinates $(p, 0)$, where $0 < p < \frac{\pi}{2}$.

(a) Find an expression, in terms of p, for the area of this rectangle.

(2)

The maximum area of $ABCD$ is S and occurs when $p = \alpha$. Show that

(b) $\frac{\pi}{4} < \alpha < 1$,

(6)

(c) $S = \frac{2\alpha^2}{\sqrt{(1+\alpha^2)}}$,

(2)

(d) $\frac{\pi^2}{2\sqrt{(16+\pi^2)}} < S < \sqrt{2}$.

(3)