1. (a) Write down the binomial expansion of $\dfrac{1}{(1-y)^2}$, $|y| < 1$, in ascending powers of y up to and including the term in y^3.

(1)

(b) Hence, or otherwise, show that

$$\frac{1}{4} \operatorname{cosec}^4\left(\frac{\theta}{2}\right) = 1 + 2\cos\theta + 3\cos^2\theta + 4\cos^3\theta + \ldots + (r+1)\cos^r\theta + \ldots$$

and state the values of θ for which this result is not valid.

(4)

Find

(c) $$1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots + \frac{(r+1)}{2^r} + \ldots,$$

(2)

(d) $$1 - \frac{2}{2} + \frac{3}{2^2} - \frac{4}{2^3} + \ldots + (-1)^r \frac{(r+1)}{2^r} + \ldots .$$

(2)