

5.

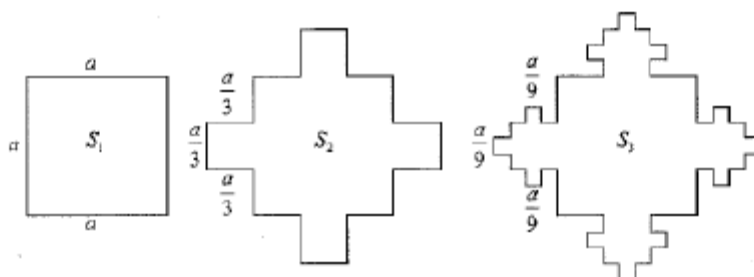


Figure 1

Figure 1 shows part of a sequence S_1, S_2, S_3, \dots , of model snowflakes. The first term S_1 consists of a single square of side a . To obtain S_2 , the middle third of each edge is replaced with a new square, of side $\frac{a}{3}$, as shown in Figure 1. Subsequent terms are obtained by replacing the middle third of each external edge of a new square formed in the previous snowflake, by a square $\frac{1}{3}$ of the size, as illustrated by S_3 in Figure 1.

- (a) Deduce that to form S_4 , 36 new squares of side $\frac{a}{27}$ must be added to S_3 . (1)
- (b) Show that the perimeters of S_2 and S_3 are $\frac{20a}{3}$ and $\frac{28a}{3}$ respectively. (2)
- (c) Find the perimeter of S_n . (4)
- (d) Describe what happens to the perimeter of S_n as n increases. (1)
- (e) Find the areas of S_1, S_2 and S_3 . (2)
- (f) Find the smallest value of the constant S such that the area of $S_n < S$, for all values of n . (5)