6. **Figure 2**

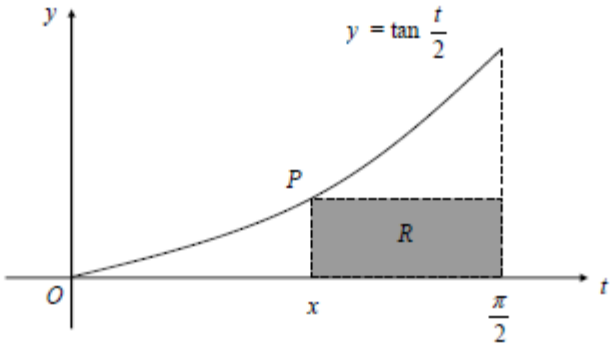

Figure 2 shows a sketch of the curve C with equation $y = \tan\frac{t}{2}$, $0 \leq t \leq \frac{\pi}{2}$.

The point P on C has coordinates $\left(x, \tan\frac{x}{2}\right)$.

The vertices of rectangle R are at $(x, 0)$, $\left(\frac{x}{2}, 0\right)$, $\left(\frac{x}{2}, \tan\frac{x}{2}\right)$ and $\left(x, \tan\frac{x}{2}\right)$ as shown in Figure 2.

(*a*) Find an expression, in terms of x, for the area A of R.

(1)

(*b*) Show that $\frac{dA}{dx} = \frac{1}{4}(\pi - 2x - 2\sin x)\sec^2\frac{x}{2}$.

(4)

(*c*) Prove that the maximum value of A occurs when $\frac{\pi}{4} < x < \frac{\pi}{3}$.

(7)

(*d*) Prove that $\tan\frac{\pi}{8} = \sqrt{2} - 1$.

(3)

(*e*) Show that the maximum value of $A > \frac{\pi}{4}(\sqrt{2} - 1)$.

(2)