

7. The points O , P and Q lie on a circle C with diameter OQ . The position vectors of P and Q , relative to O , are \mathbf{p} and \mathbf{q} respectively.

(a) Prove that $\mathbf{p} \cdot \mathbf{q} = |\mathbf{p}|^2$.

(3)

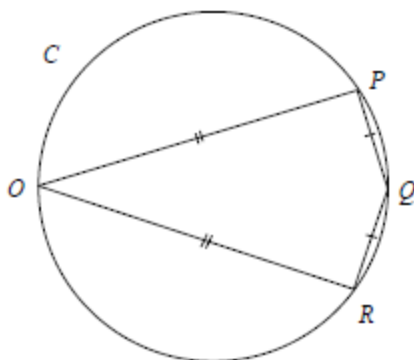


Figure 3

The point R also lies on C and $OPQR$ is a kite K as shown in Figure 3. The point S has position vector, relative to O , of $\lambda\mathbf{q}$, where λ is a constant. Given that $\mathbf{p} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{q} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and that OQ is perpendicular to PS , find

(b) the value of λ ,

(2)

(c) the position vector of R ,

(3)

(d) the area of K .

(4)

Another circle C_1 is drawn inside K so that the 4 sides of the kite are each tangents to C_1 .

(e) Find the radius of C_1 giving your answer in the form $(\sqrt{2}-1)\sqrt{n}$, where n is an integer.

(5)

A second kite K_1 is similar to K and is drawn inside C_1 .

(f) Find that area of K_1 .

(3)