4. (a) The function f(x) has f$'(x) = \dfrac{u(x)}{v(x)}$. Given that f$'(k) = 0$,

show that f$''(k) = \dfrac{u'(k)}{v(k)}$.

(3)

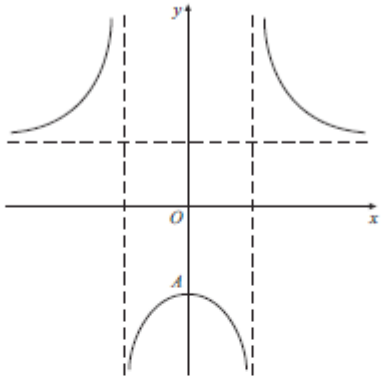

Figure 1

(b) The curve C with equation

$$y = \frac{2x^2 + 3}{x^3 - 1}$$

crosses the y-axis at the point A. Figure 1 shows a sketch of C together with its 3 asymptotes.

(i) Find the coordinates of the point A.

(1)

(ii) Find the equations of the asymptotes of C.

(2)

The point $P(a, b)$, $a > 0$ and $b > 0$, lies on C. The point Q also lies on C with PQ parallel to the x-axis and $AP = AQ$.

(iii) Show that the area of triangle PAQ is given by $\dfrac{5a^3}{a^3 - 1}$.

(2)

(iv) Find, as a varies, the minimum area of triangle PAQ, giving your answer in its simplest form.

(6)