6.

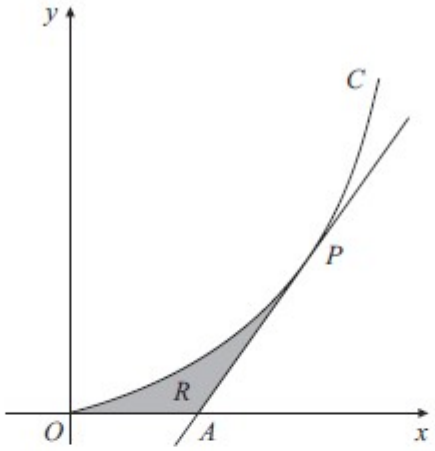

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\sin t, \qquad y = \ln(\sec t), \qquad 0 \leqslant t < \frac{\pi}{2}.$$

The tangent to C at the point P, where $t = \frac{\pi}{3}$, cuts the x-axis at A.

(a) Show that the x-coordinate of A is $\frac{\sqrt{3}}{3}(3 - \ln 2)$.

(6)

The shaded region R lies between C, the positive x-axis and the tangent AP as shown in Figure 2.

(b) Show that the area of R is $\sqrt{3}(1 + \ln 2) - 2\ln(2 + \sqrt{3}) - \frac{\sqrt{3}}{6}(\ln 2)^2$.

(11)