

7. Relative to a fixed origin O the points A , B and C have position vectors

$$\mathbf{a} = -\mathbf{i} + \frac{4}{3}\mathbf{j} + 7\mathbf{k}, \quad \mathbf{b} = 4\mathbf{i} + \frac{4}{3}\mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \mathbf{c} = 6\mathbf{i} + \frac{16}{3}\mathbf{j} + 2\mathbf{k} \quad \text{respectively.}$$

(a) Find the cosine of angle ABC .

(3)

The quadrilateral $ABCD$ is a kite K .

(b) Find the area of K .

(3)

A circle is drawn inside K so that it touches each of the 4 sides of K .

(c) Find the radius of the circle, giving your answer in the form $p\sqrt{q} - q\sqrt{p}$, where p and q are positive integers.

(5)

(d) Find the position vector of the point D .

(7)