4. The curve C has parametric equations

$$x = \cos^2 t$$

$$y = \cos t \sin t$$

where $0 \leqslant t < \pi$

(a) Show that C is a circle and find its centre and its radius.

(5)

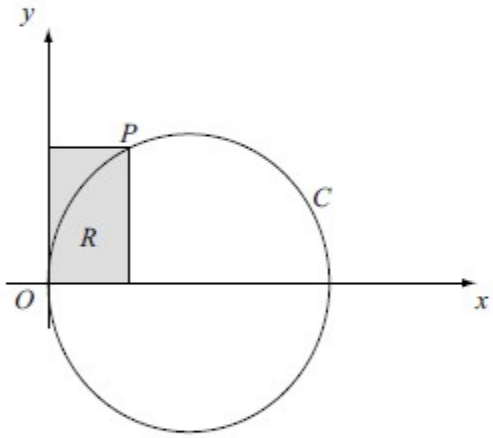

Figure 1

Figure 1 shows a sketch of C. The point P, with coordinates $\left(\cos^2 \alpha, \cos\alpha \sin \alpha\right)$, $0 < \alpha < \frac{\pi}{2}$, lies on C. The rectangle R has one side on the x-axis, one side on the y-axis and OP as a diagonal, where O is the origin.

(b) Show that the area of R is $\sin \alpha \cos^3 \alpha$

(1)

(c) Find the maximum area of R, as α varies.

(7)