5.

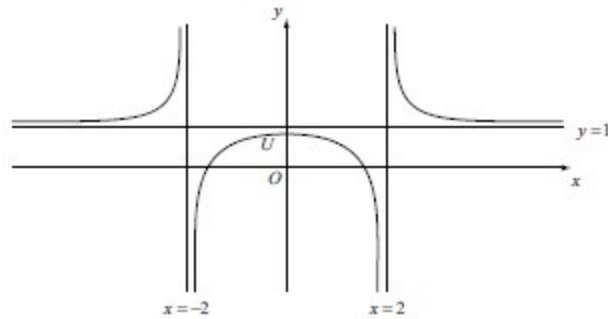**Figure 2**

Figure 2 shows a sketch of the curve C with equation $y = \dfrac{x^2 - 2}{x^2 - 4}$ and $x \neq \pm 2$.

The curve cuts the y-axis at U.

(a) Write down the coordinates of the point U.

(1)

The point P with x-coordinate a $(a \neq 0)$ lies on C.

(b) Show that the normal to C at P cuts the y-axis at the point

$$\left(0, \left[\frac{a^2 - 2}{a^2 - 4} - \frac{\left(a^2 - 4\right)^2}{4} \right] \right)$$

(6)

The circle E, with centre on the y-axis, touches all three branches of C.

(c) (i) Show that

$$\left[\frac{a^2}{2(a^2 - 4)} - \frac{\left(a^2 - 4\right)^2}{4} \right]^2 = a^2 + \frac{\left(a^2 - 4\right)^4}{16}$$

(ii) Hence, show that

$$\left(a^2 - 4\right)^2 = 1$$

(iii) Find the centre and radius of E.

(10)