$$\mathbf{a} = \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 5 \\ -2 \\ 9 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 8 \\ -4 \\ 3 \end{pmatrix}$$

The points A, B and C with position vectors a, b and c, respectively, are 3 vertices of a cube.

(a) Find the volume of the cube.

(5)

The points P, Q and R are vertices of a second cube with $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ 4 \\ \alpha \end{pmatrix}$, $\overrightarrow{PR} = \begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix}$ and α a positive constant.

(b) Given that angle $QPR = 60^{\circ}$, find the value of α .

(3)

(c) Find the length of a diagonal of the second cube.

(3)