7. [arccos$x$ and arctan$x$ are alternative notation for $\cos^{-1}x$ and $\tan^{-1}x$ respectively]

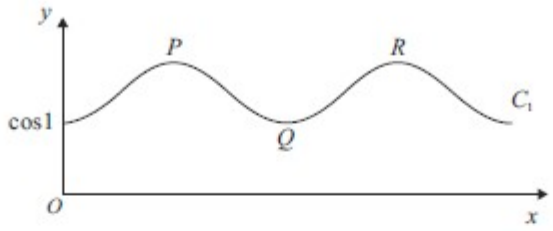

Figure 2

Figure 2 shows a sketch of the curve C_1 with equation $y = \cos(\cos x)$, $0 \leqslant x < 2\pi$.

The curve has turning points at $(0, \cos 1)$, P, Q and R as shown in Figure 2.

(a) Find the coordinates of the points P, Q and R.

(4)

The curve C_2 has equation $y = \sin(\cos x)$, $0 \leqslant x < 2\pi$. The curves C_1 and C_2 intersect at the points S and T.

(b) Copy Figure 2 and on this diagram sketch C_2 stating the coordinates of the minimum point on C_2 and the points where C_2 meets or crosses the coordinate axes.

(5)

The coordinates of S are (α, d) where $0 < \alpha < \pi$.

(c) Show that $\alpha = \arccos\left(\frac{\pi}{4}\right)$.

(2)

(d) Find the value of d in surd form and write down the coordinates of T.

(3)

The tangent to C_1 at the point S has gradient $\tan\beta$.

(e) Show that $\beta = \arctan\sqrt{\left(\frac{16-\pi^2}{32}\right)}$.

(5)

(f) Find, in terms of β, the obtuse angle between the tangent to C_1 at S and the tangent to C_2 at S.

(5)