5. In this question u and v are functions of x. Given that $\int u\,dx$, $\int v\,dx$ and $\int uv\,dx$ satisfy

$$\int uv\,dx = \left(\int u\,dx\right) \times \left(\int v\,dx\right) \qquad uv \neq 0$$

(a) show that $1 = \dfrac{\int u\,dx}{u} + \dfrac{\int v\,dx}{v}$

(3)

Given also that $\dfrac{\int u\,dx}{u} = \sin^2 x$,

(b) use part (a) to write down an expression, in terms of x, for $\dfrac{\int v\,dx}{v}$,

(1)

(c) show that

$$\frac{1}{u}\frac{du}{dx} = \frac{1 - 2\sin x \cos x}{\sin^2 x}$$

(3)

(d) hence use integration to show that $u = Ae^{-\cot x}\operatorname{cosec}^2 x$, where A is an arbitrary constant.

(6)

(e) By differentiating $e^{\tan x}$ find a similar expression for v.

(2)