7.

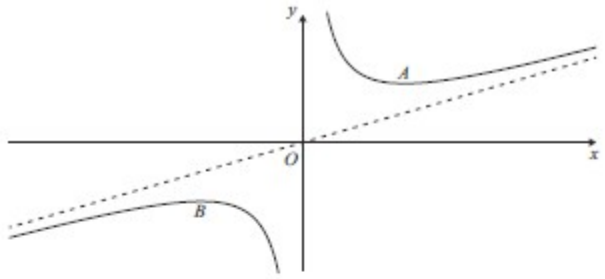

Figure 1

Figure 1 shows a sketch of the curve C_1 with equation $y = f(x)$ where

$$f(x) = \frac{x}{3} + \frac{12}{x} \qquad x \neq 0$$

The lines $x = 0$ and $y = \frac{x}{3}$ are asymptotes to C_1. The point A on C_1 is a minimum and the point B on C_1 is a maximum.

(a) Find the coordinates of A and B.

(4)

There is a normal to C_1, which does not intersect C_1 a second time, that has equation $x = k$, where $k > 0$.

(b) Write down the value of k.

(1)

The point $P(\alpha, \beta)$, $\alpha > 0$ and $\alpha \neq k$, lies on C_1. The normal to C_1 at P does not intersect C_1 a second time.

(c) Find the value of α, leaving your answer in simplified surd form.

(5)

(d) Find the equation of this normal.

(3)

The curve C_2 has equation $y = |f(x)|$

(e) Sketch C_2 stating the coordinates of any turning points and the equations of any asymptotes.

(4)

The line with equation $y = mx + 1$ does not touch or intersect C_2.

(f) Find the set of possible values for m.

(5)