4. Given that

$$(1+x)^n = 1 + \sum_{r=1}^{\infty} \frac{n(n-1)\dots(n-r+1)}{1\times 2\times \dots \times r}x^r \qquad (|x|<1, x\in\mathbb{R}, n\in\mathbb{R})$$

(a) show that

$$(1-x)^{-\frac{1}{2}} = \sum_{r=0}^{\infty}\binom{2r}{r}\left(\frac{x}{4}\right)^r$$

(5)

(b) show that $(9-4x^2)^{-\frac{1}{2}}$ can be written in the form $\sum_{r=0}^{\infty}\binom{2r}{r}\frac{x^{2r}}{3^q}$ and give q in terms of r.

(3)

(c) Find $\sum_{r=1}^{\infty}\binom{2r}{r}\times\frac{2r}{9}\times\left(\frac{x}{3}\right)^{2r-1}$

(3)

(d) Hence find the exact value of

$$\sum_{r=1}^{\infty}\binom{2r}{r}\times\frac{2r\sqrt{5}}{9}\times\frac{1}{5^r}$$

giving your answer as a rational number.

(2)