6. (i) A curve with equation $y = f(x)$ has $f(x) \geqslant 0$ for $x \geqslant a$ and

$$A = \int_a^b f(x)\,dx \quad \text{and} \quad V = \pi \int_a^b [f(x)]^2\,dx$$

where a and b are constants with $b > a$.

Use integration by substitution to show that for the positive constants r and h

$$\pi \int_{a+h}^{b+h} [r + f(x-h)]^2\,dx = \pi r^2 (b-a) + 2\pi r A + V$$

(3)

(ii)

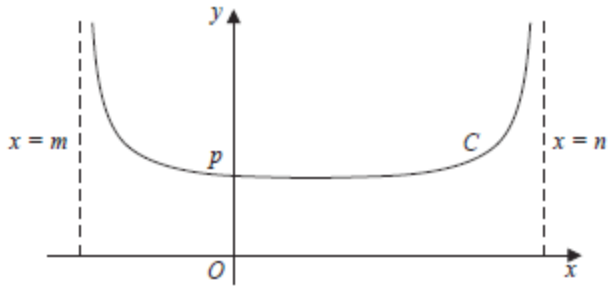

Figure 1

Figure 1 shows part of the curve C with equation $y = 4 + \dfrac{2}{\sqrt{3}\cos x + \sin x}$

This curve has asymptotes $x = m$ and $x = n$ and crosses the y-axis at $(0, p)$.

(a) Find the value of p, the value of m and the value of n.

(4)

(b) Show that the equation of C can be written in the form $y = r + f(x - h)$ and specify the function f and the constants r and h.

(4)

The region bounded by C, the x-axis and the lines $x = \dfrac{\pi}{6}$ and $x = \dfrac{\pi}{3}$ is rotated through 2π radians about the x-axis.

(c) Find the volume of the solid formed.

(9)