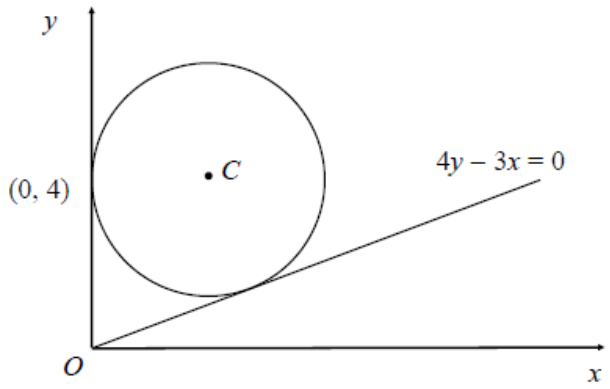
Core 2 Integration Questions
(From AEA Papers)

For answers, see the AEA website

2004, Question 4:

4. **Figure 1**



The circle, with centre C and radius r, touches the y-axis at $(0, 4)$ and also touches the line with equation $4y - 3x = 0$, as shown in Fig. 1.

(*a*) (i) Find the value of r.

(ii) Show that $\arctan\left(\frac{3}{4}\right) + 2\arctan\left(\frac{1}{2}\right) = \frac{1}{2}\pi$.

(8)

The line with equation $4x + 3y = q$, $q > 12$, is a tangent to the circle.

(*b*) Find the value of q.

(4)

2007, Question 2:

2. (*a*) On the same diagram, sketch $y = x$ and $y = \sqrt{x}$, for $x \geq 0$, and mark clearly the coordinates of the points of intersection of the two graphs.

(2)

(*b*) With reference to your sketch, explain why there exists a value a of x ($a > 1$) such that

$$\int_0^a x \, dx = \int_0^a \sqrt{x} \, dx .$$

(2)

(*c*) Find the exact value of a.

(4)

(*d*) Hence, or otherwise, find a non-constant function f(x) and a constant b ($b \neq 0$) such that

$$\int_{-b}^b \mathrm{f}(x) \, dx = \int_{-b}^b \sqrt{[\mathrm{f}(x)]} \, dx .$$

(2)

2012, Question 6:

6.

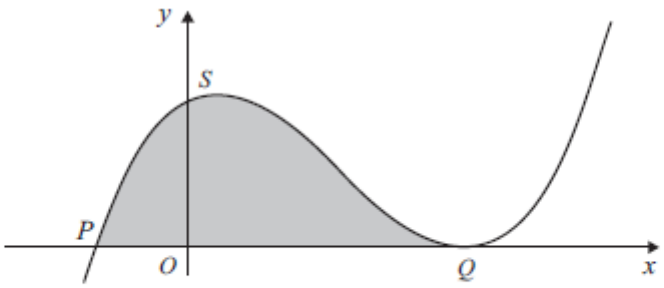


Figure 1

Figure 1 shows a sketch of the curve with equation $y = (x + a)(x - b)^2$, where a and b are positive constants. The curve cuts the x-axis at P and has a maximum point at S and a minimum point at Q.

(a) Write down the coordinates of P and Q in terms of a and b.

(2)

(b) Show that G, the area of the shaded region between the curve PSQ and the x-axis, is given by $G = \dfrac{(a+b)^4}{12}$.

(6)

The rectangle $PQRST$ has RST parallel to QP and both PT and QR are parallel to the y-axis.

(c) Show that $\dfrac{G}{\text{Area of } PQRST} = k$, where k is a constant independent of a and b and find the value of k.

(8)