Core 3 Functions Questions
(From AEA Papers)

For answers, see the AEA website

2003, Question 5:

5. The function f is given by

$$f(x) = \frac{1}{\lambda}(x^2 - 4)(x^2 - 25),$$

where x is real and λ is a positive integer.

(a) Sketch the graph of $y = f(x)$ showing clearly where the graph crosses the coordinate axes.

(3)

(b) Find, in terms of λ, the range of f.

(5)

(c) Find the sets of positive integers k and λ such that the equation

$$k = |f(x)|$$

has exactly k distinct real roots.

(9)

2005, Question 6:

6. **Figure 1**

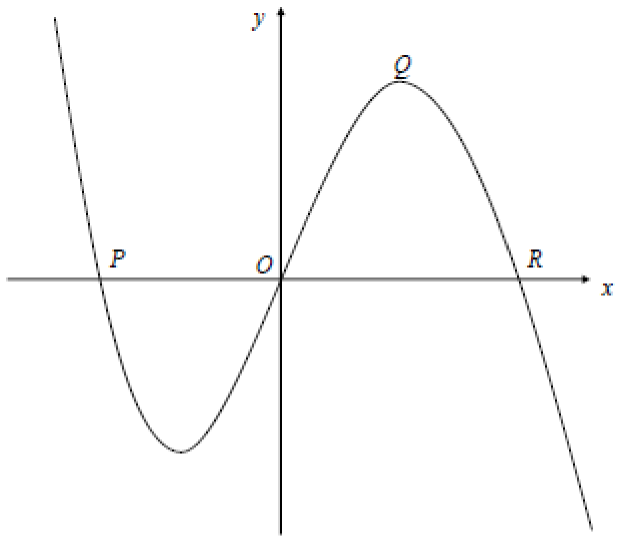

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = x(12 - x^2).$$

The curve cuts the x-axis at the points P, O and R, and Q is the maximum point.

(a) Find the coordinates of the points P, Q and R.

(4)

(b) Sketch, on separate diagrams, the graphs of

(i) $y = f(2x)$,

(ii) $y = f(|x| + 1)$,

indicating on each sketch the coordinates of any maximum points and the intersections with the x-axis.

(6)

Figure 2

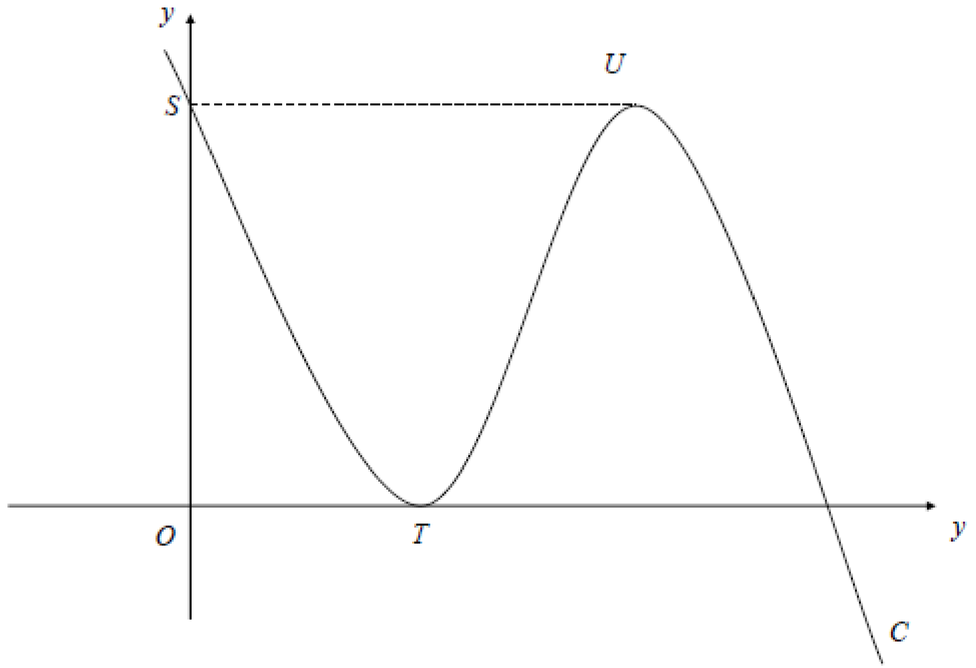

Figure 2 shows a sketch of part of the curve C, with equation $y = \text{f}(x - v) + w$, where v and w are constants. The x-axis is a tangent to C at the minimum point T, and C intersects the y-axis at S. The line joining S to the maximum point U is parallel to the x-axis.

(c) Find the value of v and the value of w and hence find the roots of the equation

$$\text{f}(x - v) + w = 0.$$

(9)

2008, Question 6:

6.

$$f(x) = \frac{ax+b}{x+2}\,; \quad x \in \mathbb{R}, x \neq -2,$$

where a and b are constants and $b > 0$.

(a) Find $f^{-1}(x)$.

(2)

(b) Hence, or otherwise, find the value of a so that $ff(x) = x$.

(2)

The curve C has equation $y = f(x)$ and $f(x)$ satisfies $ff(x) = x$.

(c) On separate axes sketch

(i) $y = f(x)$,

(3)

(ii) $y = f(x-2) + 2$.

(3)

On each sketch you should indicate the equations of any asymptotes and the coordinates, in terms of b, of any intersections with the axes.

The normal to C at the point P has equation $y = 4x - 39$. The normal to C at the point Q has equation $y = 4x + k$, where k is a constant.

(d) By considering the images of the normals to C on the curve with equation $y = f(x-2) + 2$, or otherwise, find the value of k.

(5)

2009, Question 1:

1. (a) On the same diagram, sketch

$$y = (x+1)(2-x) \quad \text{and} \quad y = x^2 - 2|x|.$$

Mark clearly the coordinates of the points where these curves cross the coordinate axes.

(3)

(b) Find the x-coordinates of the points of intersection of these two curves.

(5)

2012, Question 1:

1. The function f is given by

$$f(x) = x^2 - 2x + 6, \qquad x \geqslant 0$$

(a) Find the range of f.

(3)

The function g is given by

$$g(x) = 3 + \sqrt{(x + 4)}, \qquad x \geqslant 2$$

(b) Find $gf(x)$.

(2)

(c) Find the domain and range of gf.

(3)

2013, Question 7:

7.

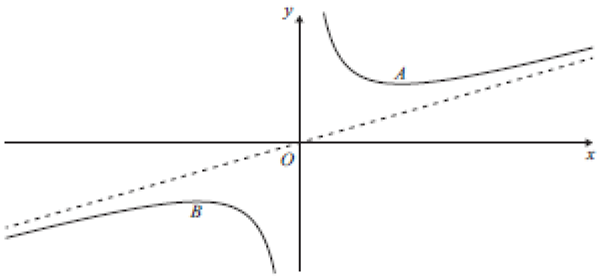

Figure 1

Figure 1 shows a sketch of the curve C_1 with equation $y = f(x)$ where

$$f(x) = \frac{x}{3} + \frac{12}{x} \qquad x \neq 0$$

The lines $x = 0$ and $y = \frac{x}{3}$ are asymptotes to C_1. The point A on C_1 is a minimum and the point B on C_1 is a maximum.

(a) Find the coordinates of A and B.

(4)

There is a normal to C_1, which does not intersect C_1 a second time, that has equation $x = k$, where $k > 0$.

(b) Write down the value of k.

(1)

The point $P(\alpha, \beta)$, $\alpha > 0$ and $\alpha \neq k$, lies on C_1. The normal to C_1 at P does not intersect C_1 a second time.

(c) Find the value of α, leaving your answer in simplified surd form.

(5)

(d) Find the equation of this normal.

(3)

The curve C_2 has equation $y = |f(x)|$

(e) Sketch C_2 stating the coordinates of any turning points and the equations of any asymptotes.

(4)

The line with equation $y = mx + 1$ does not touch or intersect C_2.

(f) Find the set of possible values for m.

(5)

2014, Question 1:

1. The function f is given by

$$f(x) = \ln(2x - 5), \qquad x > 2.5$$

(a) Find $f^{-1}(x)$.

(2)

The function g has domain $x > 2$ and

$$fg(x) = \ln\left(\frac{x+10}{x-2}\right), \qquad x > 2$$

(b) Find $g(x)$ and simplify your answer.

(3)

2014, Question 3:

3. (a) On separate diagrams sketch the curves with the following equations. On each sketch you should mark the coordinates of the points where the curve crosses the coordinate axes.

(i) $y = x^2 - 2x - 3$

(ii) $y = x^2 - 2|x| - 3$

(iii) $y = x^2 - x - |x| - 3$

(7)

(b) Solve the equation

$$x^2 - x - |x| - 3 = x + |x|$$

(4)

2014, Question 6:

6. (i) A curve with equation $y = \mathrm{f}(x)$ has $\mathrm{f}(x) \geqslant 0$ for $x \geqslant a$ and

$$A = \int_a^b \mathrm{f}(x)\,\mathrm{d}x \quad \text{and} \quad V = \pi\int_a^b [\mathrm{f}(x)]^2\,\mathrm{d}x$$

where a and b are constants with $b > a$.

Use integration by substitution to show that for the positive constants r and h

$$\pi\int_{a+h}^{b+h} [r + \mathrm{f}(x-h)]^2\,\mathrm{d}x = \pi r^2(b-a) + 2\pi rA + V$$

(3)

(ii)

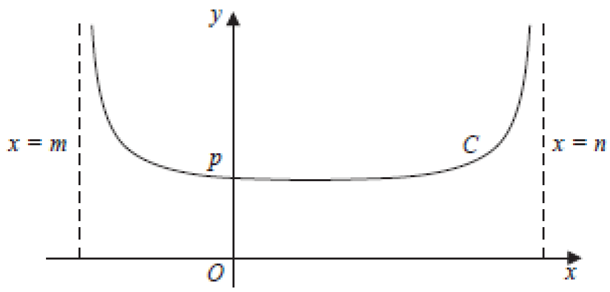

Figure 1

Figure 1 shows part of the curve C with equation $y = 4 + \dfrac{2}{\sqrt{3}\cos x + \sin x}$

This curve has asymptotes $x = m$ and $x = n$ and crosses the y-axis at $(0, p)$.

(a) Find the value of p, the value of m and the value of n.

(4)

(b) Show that the equation of C can be written in the form $y = r + \mathrm{f}(x - h)$ and specify the function f and the constants r and h.

(4)

The region bounded by C, the x-axis and the lines $x = \dfrac{\pi}{6}$ and $x = \dfrac{\pi}{3}$ is rotated through 2π radians about the x-axis.

(c) Find the volume of the solid formed.

(9)