Let $f(n)$ be a function defined, for any integer $n \geqslant 0$, as follows:

$$f(n) = \begin{cases} 1 & \text{if } n = 0, \\ (f(n/2))^2 & \text{if } n > 0 \text{ and } n \text{ is even}, \\ 2f(n-1) & \text{if } n > 0 \text{ and } n \text{ is odd}. \end{cases}$$

(i) What is the value of $f(5)$?

The *recursion depth* of $f(n)$ is defined to be the number of other integers m such that the value of $f(m)$ is calculated whilst computing the value of $f(n)$. For example, the recursion depth of $f(4)$ is 3, because the values of $f(2)$, $f(1)$, and $f(0)$ need to be calculated on the way to computing the value of $f(4)$.

(ii) What is the recursion depth of $f(5)$?

Now let $g(n)$ be a function, defined for all integers $n \geqslant 0$, as follows:

$$g(n) = \begin{cases} 0 & \text{if } n = 0, \\ 1 + g(n/2) & \text{if } n > 0 \text{ and } n \text{ is even}, \\ 1 + g(n-1) & \text{if } n > 0 \text{ and } n \text{ is odd}. \end{cases}$$

(iii) What is $g(5)$?

(iv) What is $g\left(2^k\right)$, where $k \geqslant 0$ is an integer? Briefly explain your answer.

(v) What is $g\left(2^l + 2^k\right)$ where $l > k \geqslant 0$ are integers? Briefly explain your answer.

(vi) Explain briefly why the value of $g(n)$ is equal to the recursion depth of $f(n)$.