(i) Find a pair of positive integers, x_1 and y_1, that solve the equation

$$(x_1)^2 - 2(y_1)^2 = 1.$$

(ii) Given integers a, b, we define two sequences $x_1, x_2, x_3, \ldots$ and $y_1, y_2, y_3, \ldots$ by setting

$$x_{n+1} = 3x_n + 4y_n, \qquad y_{n+1} = ax_n + by_n, \qquad \text{for } n \geqslant 1.$$

Find *positive* values for a, b such that

$$(x_{n+1})^2 - 2(y_{n+1})^2 = (x_n)^2 - 2(y_n)^2.$$

(iii) Find a pair of integers X, Y which satisfy $X^2 - 2Y^2 = 1$ such that $X > Y > 50$.

(iv) (Using the values of a and b found in part (ii)) what is the approximate value of x_n/y_n as n increases?