Ox-words are sequences of letters a and b that are constructed according to the following rules:

I. The sequence consisting of no letters is an Ox-word.

II. If the sequence W is an Ox-word, then the sequence that begins with a, followed by W and ending in b, written aWb, is an Ox-word.

III. If the sequences U and V are Ox-words, then the sequence U followed by V, written UV, is an Ox-word.

All Ox-words are constructed using these rules. The *length* of an Ox-word is the number of letters that occur in it. For example *aabb* and *abab* are Ox-words of length 4.

(i) Show that every Ox-word has an even length.

(ii) List all Ox-words of length 6.

(iii) Let W be an Ox-word. Is the number of occurrences of a in W necessarily equal to the number of occurrences of b in W? Justify your answer.

You may now assume that every Ox-word (of positive length) can be written *uniquely* in the form $aWbW'$ where W and W' are Ox-words.

(iv) For $n \geqslant 0$, let C_n be the number of Ox-words of length $2n$. Find an expression for C_{n+1} in terms of $C_0, C_1, \cdots, C_n$. Explain your reasoning.