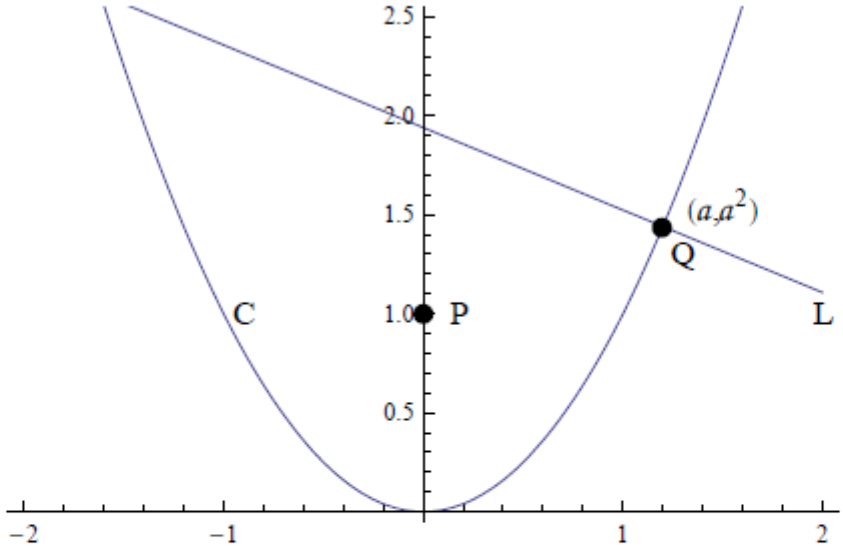
As shown in the diagram below: C is the parabola with equation $y = x^2$; P is the point $(0, 1)$; Q is the point (a, a^2) on C; L is the normal to C which passes through Q.



(i) Find the equation of L.

(ii) For what values of a does L pass through P?

(iii) Determine $|QP|^2$ as a function of a, where $|QP|$ denotes the distance from P to Q.

(iv) Find the values of a for which $|QP|$ is smallest.

(v) Find a point R, in the xy-plane but not on C, such that $|RQ|$ is smallest for a unique value of a. Briefly justify your answer.