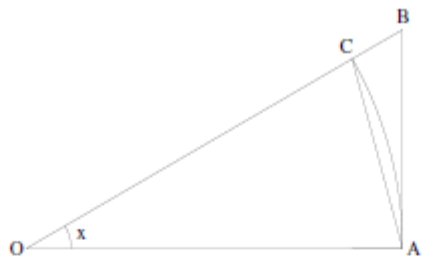


[In this question, you may assume that the derivative of  $\sin x$  is  $\cos x$ .]



(i) In the diagram above  $OA$  and  $OC$  are of length 1 and subtend an angle  $x$  at  $O$ . The angle  $BAO$  is a right angle and the circular arc from  $A$  to  $C$ , centred at  $O$ , is also drawn.

By consideration of various areas in the above diagram, show, for  $0 < x < \pi/2$ , that

$$x \cos x < \sin x < x.$$

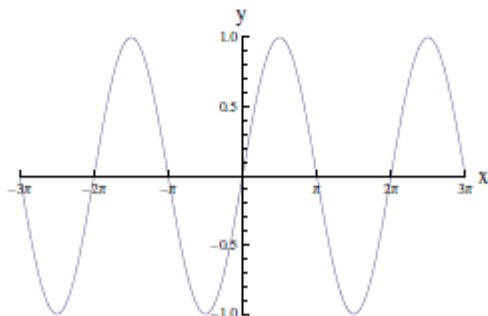
(ii) Sketch, on the axes provided on the opposite page, the graph of

$$y = \frac{\sin x}{x}, \quad 0 < x < 4\pi.$$

Justify your value that  $y$  takes as  $x$  becomes small.

[You do not need to determine the coordinates of the turning points.]

(iii) Drawn below is a graph of  $y = \sin x$ . Sketch on the same axes the line  $y = cx$  where  $c > 0$  is such that the equation  $\sin x = cx$  has *exactly 5 solutions*.



(iv) Draw the line  $y = c$  on the axes on the opposite page.

(v) If  $X$  is the largest of the five solutions of the equation  $\sin x = cx$ , explain why  $\tan X = X$ .