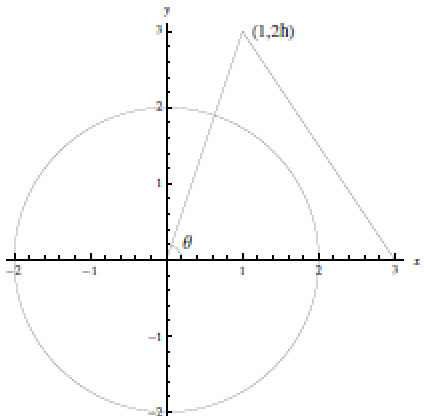

Diagram when $h > 2/\sqrt{5}$

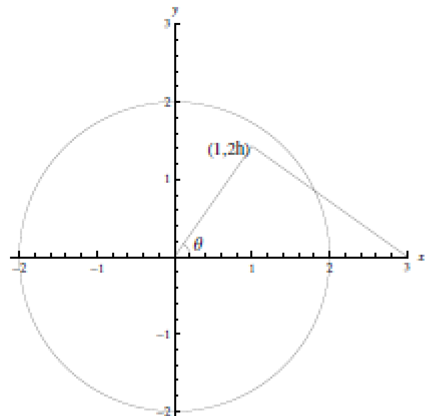

Diagram when $h < \sqrt{3}/2$

The three corners of a triangle T are $(0,0)$, $(3,0)$, $(1,2h)$ where $h > 0$. The circle C has equation $x^2 + y^2 = 4$. The angle of the triangle at the origin is denoted as θ. The circle and triangle are drawn in the diagrams above for different values of h.

(i) Express $\tan\theta$ in terms of h.

(ii) Show that the point $(1,2h)$ lies inside C when $h < \sqrt{3}/2$.

(iii) Find the equation of the line connecting $(3,0)$ and $(1,2h)$. Show that this line is tangential to the circle C when $h = 2/\sqrt{5}$.

(iv) Suppose now that $h > 2/\sqrt{5}$. Find the area of the region inside both C and T in terms of θ.

(v) Now let $h = 6/7$. Show that the point $(8/5, 6/5)$ lies on both the line (from part (iii)) and the circle C.

Hence show that the area of the region inside both C and T equals

$$\frac{27}{35} + 2\alpha$$

where α is an angle whose tangent, $\tan\alpha$, you should determine.

[You may use the fact that the area of a triangle with corners $(0,0)$, (a,b), (c,d) equals $\frac{1}{2}\left|ad - bc\right|$.]