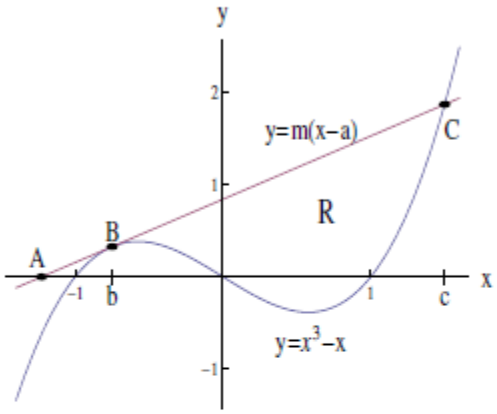
The graphs of $y = x^3 - x$ and $y = m(x - a)$ are drawn on the axes below. Here $m > 0$ and $a \leqslant -1$.

The line $y = m(x - a)$ meets the x-axis at $A = (a, 0)$, *touches* the cubic $y = x^3 - x$ at B and intersects again with the cubic at C. The x-coordinates of B and C are respectively b and c.



(i) Use the fact that the line and cubic *touch* when $x = b$, to show that $m = 3b^2 - 1$.

(ii) Show further that

$$a = \frac{2b^3}{3b^2 - 1}.$$

(iii) If $a = -10^6$, what is the approximate value of b?

(iv) Using the fact that

$$x^3 - x - m(x - a) = (x - b)^2 (x - c)$$

(which you need not prove), show that $c = -2b$.

(v) R is the finite region bounded above by the line $y = m(x - a)$ and bounded below by the cubic $y = x^3 - x$. For what value of a is the area of R largest?

Show that the largest possible area of R is $\frac{27}{4}$.