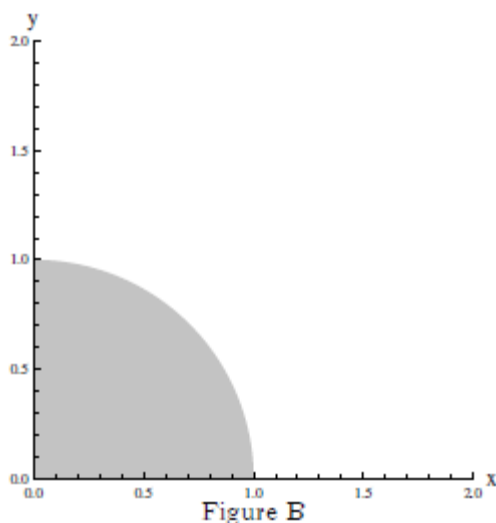
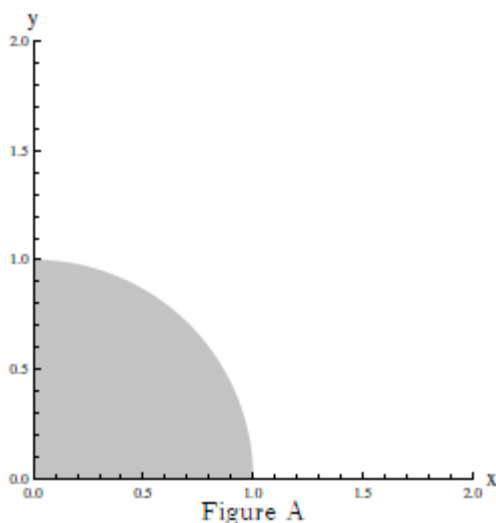


Let Q denote the quarter-disc of points (x, y) such that $x \geq 0$, $y \geq 0$ and $x^2 + y^2 \leq 1$ as drawn in Figures A and B below.



(i) On the axes in Figure A, sketch the graphs of

$$x + y = \frac{1}{2}, \quad x + y = 1, \quad x + y = \frac{3}{2}.$$

What is the largest value of $x + y$ achieved at points (x, y) in Q ? Justify your answer.

(ii) On the axes in Figure B, sketch the graphs of

$$xy = \frac{1}{4}, \quad xy = 1, \quad xy = 2.$$

What is the largest value of $x^2 + y^2 + 4xy$ achieved at points (x, y) in Q ?

What is the largest value of $x^2 + y^2 - 6xy$ achieved at points (x, y) in Q ?

(iii) Describe the curve

$$x^2 + y^2 - 4x - 2y = k$$

where $k > -5$.

What is the *smallest* value of $x^2 + y^2 - 4x - 2y$ achieved at points (x, y) in Q ?