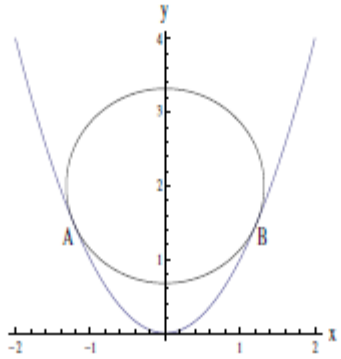
The diagram below shows the parabola $y = x^2$ and a circle with centre $(0, 2)$ just 'resting' on the parabola. By 'resting' we mean that the circle and parabola are tangential to each other at the points $A$ and $B$.




(i) Let $(x, y)$ be a point on the parabola such that $x \neq 0$. Show that the gradient of the line joining this point to the centre of the circle is given by

$$\frac{x^2 - 2}{x}.$$

(ii) With the help of the result from part (i), or otherwise, show that the coordinates of $B$ are given by

$$\left(\sqrt{\frac{3}{2}}, \frac{3}{2}\right).$$

(iii) Show that the area of the sector of the circle enclosed by the radius to $A$, the minor arc $AB$ and the radius to $B$ is equal to

$$\frac{7}{4}\cos^{-1}\left(\frac{1}{\sqrt{7}}\right).$$

(iv) Suppose now that a circle with centre $(0, a)$ is resting on the parabola, where $a > 0$. Find the range of values of $a$ for which the circle and parabola touch at two distinct points.

(v) Let $r$ be the radius of a circle with centre $(0, a)$ that is resting on the parabola. Express $a$ as a function of $r$, distinguishing between the cases in which the circle is, and is not, in contact with the vertex of the parabola.