A particular robot has three commands:

F: Move forward a unit distance;
L: Turn left 90°;
R: Turn right 90°.

A *program* is a sequence of commands. We consider particular programs P_n (for $n \geqslant 0$) in this question. The basic program P_0 just instructs the robot to move forward:

$$P_0 = \mathbf{F}.$$

The program P_{n+1} (for $n \geqslant 0$) involves performing P_n, turning left, performing P_n again, then turning right:

$$P_{n+1} = P_n \mathbf{L} P_n \mathbf{R}.$$

So, for example, $P_1 = \mathbf{F}\,\mathbf{L}\,\mathbf{F}\,\mathbf{R}$.

(i) Write down the program P_2.

(ii) How far does the robot travel during the program P_n? In other words, how many **F** commands does it perform?

(iii) Let l_n be the total number of commands in P_n; so, for example, $l_0 = 1$ and $l_1 = 4$.

Write down an equation relating l_{n+1} to l_n. Hence write down a formula for l_n in terms of n. No proof is required. **Hint:** consider $l_n + 2$.

(iv) The robot starts at the origin, facing along the positive x-axis. What direction is the robot facing after performing the program P_n?

(v) The left-hand diagram on the opposite page shows the path the robot takes when it performs the program P_1. On the right-hand diagram opposite, draw the path it takes when it performs the program P_4.

(vi) Let (x_n, y_n) be the position of the robot after performing the program P_n, so $(x_0, y_0) = (1, 0)$ and $(x_1, y_1) = (1, 1)$. Give an equation relating (x_{n+1}, y_{n+1}) to (x_n, y_n).

What is (x_8, y_8)? What is (x_{8k}, y_{8k})?