Alice, Bob and Charlie are well-known expert logicians; they always tell the truth.

In each of the scenarios below, Charlie writes a whole number on Alice and Bob's foreheads. The difference between the two numbers is one: either Alice's number is one larger than Bob's, or Bob's number is one larger than Alice's. Each of Alice and Bob can see the number on the other's forehead, but can't see their own number.

(i) Charlie writes a number on Alice and Bob's foreheads, and says "Each of your numbers is at least 1. The difference between the numbers is 1."

Alice then says "I know my number."

Explain why Alice's number must be 2. What is Bob's number?

(ii) Charlie now writes new numbers on their foreheads, and says "Each of your numbers is between 1 and 10 inclusive. The difference between the numbers is 1. Alice's number is a prime." (A prime number is a number greater than 1 that is divisible only by 1 and itself.)

Alice then says "I don't know my number."

Bob then says "I don't know my number."

What is Alice's number? Explain your answer.

(iii) Charlie now writes new numbers on their foreheads, and says "Each of your numbers is between 1 and 10 inclusive. The difference between the numbers is 1."

Alice then says "I don't know my number. Is my number a square number?"

Charlie then says "If I told you that, you would know your number."

Bob then says "I don't know my number."

What is Alice's number? Explain your answer.