Alice plays a game 5 times with her friends Sam and Pam. In each game Alice chooses two integers x and y with $1 \leqslant x \leqslant y$. She whispers the sum $x + y$ to Sam, and the product $x \times y$ to Pam, so that neither knows what the other was told. Sam and Pam then have to try to work out what the numbers x and y are. Sam and Pam are well known expert logicians.

(i) In the first game, Pam says "I know x and y."

What can we deduce about the values of x and y? Explain your answer.

(ii) In the second game, Pam says "I don't know what x and y are."

Sam then says "I know x and y."

Suppose the sum is 4. What are x and y? Explain your answer.

(iii) In the third game, Pam says "I don't know what x and y are."

Sam then says "I don't know what x and y are."

Pam then says "I now know x and y."

Suppose the product is 4. What are x and y? Explain your answer.

(iv) In the fourth game, Pam says "I don't know what x and y are."

Sam then says "I already knew that."

Pam then says "I now know x and y."

Suppose the product is 8. What are x and y? Explain your answer.

(v) Finally, in the fifth game, Pam says "I don't know what x and y are."

Sam then says "I don't know what x and y are."

Pam then says "I don't know what x and y are."

Sam then says "I now know x and y."

Suppose the sum is 5. What are x and y? Explain your answer.