15. For how many integers n is $\dfrac{n}{100 - n}$ also an integer?

A 1 B 6 C 10 D 18 E 100

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15. D Let $\dfrac{n}{100 - n} = x$ where x is an integer. Hence $n = 100x - nx$.

Hence $n(1 + x) = 100x$ giving $n = \dfrac{100x}{1 + x}$.

Now x and $1 + x$ can have no common factors. Therefore $1 + x$ must be a factor of 100 and can be any of them.

Hence $1 + x \in \{\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm 25, \pm 50, \pm 100\}$ thus the number of possible integers n is 18.