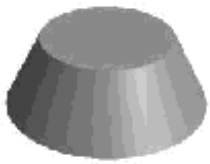
21. A frustum is the solid obtained by slicing a right-circular cone perpendicular to its axis and removing the small cone above the slice. This leaves a shape with two circular faces and a curved surface. The original cone has base radius 6 cm and height 8 cm, and the curved surface area of the frustum is equal to the area of the two circles. What is the height of the frustum?



A 3 cm B 4 cm C 5 cm D 6 cm E 7 cm

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21. B Let r be the radius of the small cone and h the height.
Let l_1 and l_2 be the slant heights of the small and large cones respectively.
By Pythagoras' Theorem $l_2 = \sqrt{6^2 + 8^2} = 10$.
Using similar triangles, $\frac{l_1}{r} = \frac{10}{6}$ so $l_1 = \frac{5}{3}r$ and $\frac{h}{8} = \frac{r}{6}$ giving $h = \frac{4}{3}r$.
Thus the area of the curved surface of the frustum is

$$\pi \times 6 \times 10 - \pi \times r \times \frac{5}{3} \times r = \pi\left(60 - \frac{5r^2}{3}\right).$$

The sum of the areas of the two circles is $\pi \times 6^2 + \pi \times r^2 = \pi(36 + r^2)$.
Hence $\pi\left(60 - \frac{5r^2}{3}\right) = \pi(36 + r^2)$ and so $24 = \frac{8r^2}{3}$ giving $r = 3$, so $h = \frac{4}{3} \times 3 = 4$.
Therefore, in cms, the height of the frustum is $8 - 4 = 4$.